Science of Nonimaging Optics: The Thermodynamic Connection

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Keynote Lecture

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UC Solar Research Innovation Education
University of California Advanced Solar Technologies Institute

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SANTA CLARA
SAN BENITO
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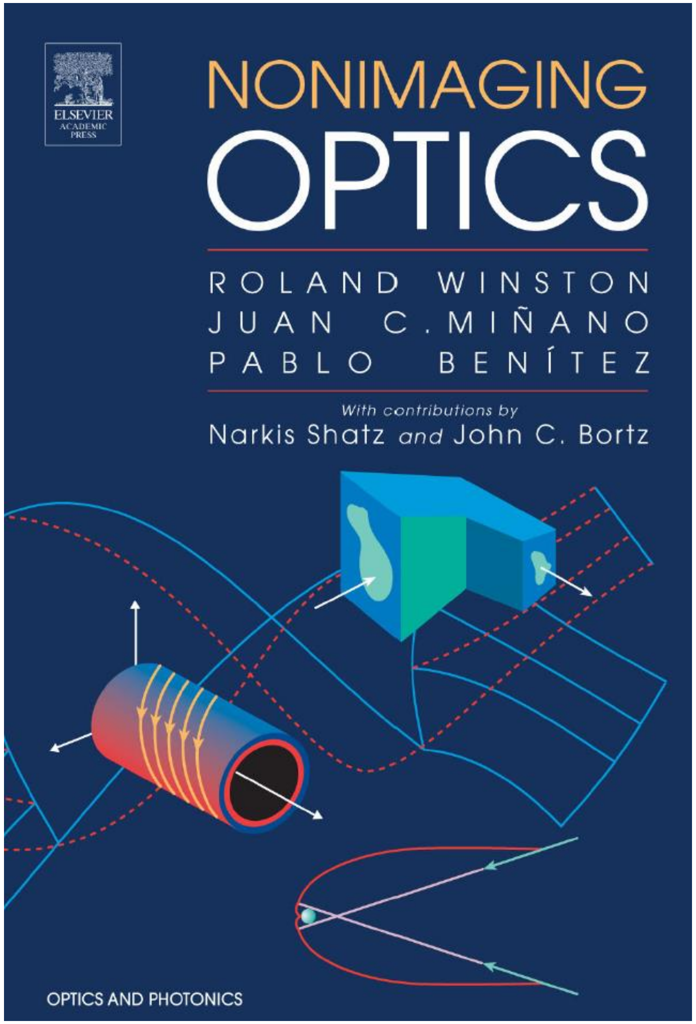
UC Merced Campus
Under Construction January 2004
Photo By Hans Marsen

The Stefan Boltzmann law σT^4

is on page 1 of an optics book---
Something interesting is going on !

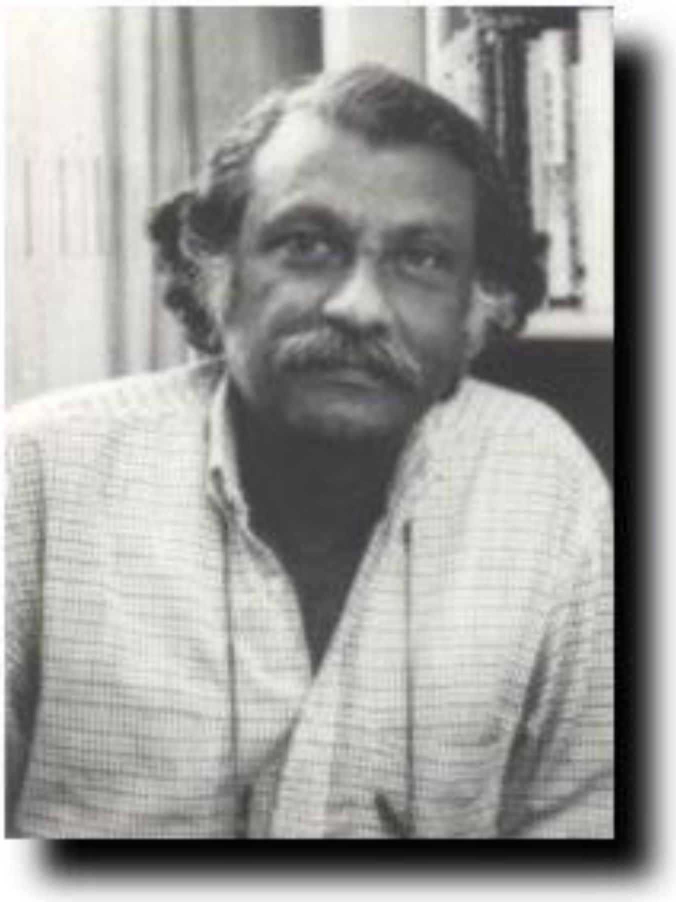


What is the best efficiency possible? When we pose this question, we are stepping outside the bounds of a particular subject. Questions of this kind are more properly the province of thermodynamics which imposes limits on the possible, like energy conservation and the impossible, like transferring heat from a cold body to a warm body without doing work.   And that is why the fusion of the science of light (optics) with the science of heat (thermodynamics), is where much of the excitement is today. During a seminar I gave some ten years ago at the Raman Institute in Bangalore, the distinguished astrophysicist Venkatraman Radhakrishnan famously asked “how come geometrical optics knows the second law of thermodynamics?” This provocative question from C. V. Raman’s son serves to frame our discussion.

During a seminar at the Raman Institute (Bangalore) in 2000,

Prof. **V. Radhakrishnan** asked me:

How does geometrical optics know the second law of thermodynamics?



A few observations suffice to establish the connection. As is well-known, the solar spectrum fits a black body at 5670 K (almost 10,000 degrees Fahrenheit) Now a black body absorbs radiation at all wavelengths, and it follows from thermodynamics that its spectrum (the Planck spectrum[1]) is uniquely specified by temperature. The well-known Stefan Boltzmann law which also follows from thermodynamics relates temperature to radiated flux so that the solar surface flux is Φs ~ 58.6 W/mm^2 while the measured flux at top of the earth's atmosphere is ^{1}.35 mW/mm^2 . That the ratio, ~ 44,000, coincides with $1/\sin^2 \Theta s$ where Θs is the angular subtense of the sun is *not* a coincidence but rather illustrates a deep connection between the two subjects (the *sine law of concentration*). *Nonimaging Optics* is the theory of thermodynamically efficient optics and as such, depends more on thermodynamics than on optics I often tell my students to learn efficient optical design, first study the *theory of furnaces*.

Invention of Entropy

(The Second Law of Thermodynamics)

- Sadi Carnot had fought with Napoleon, but by 1824 was a student studying physics in Paris. In that year he wrote:
- Reflections on the Motive Power of Heat and on Machines fitted to Develop that Power.
- The conservation of energy (the first law of thermodynamics) had not yet been discovered, heat was considered a conserved fluid-"caloric"
- So ENTROPY (the second law of thermodynamics) was discovered first.
- A discovery way more significant than all of Napoleon's conquests!

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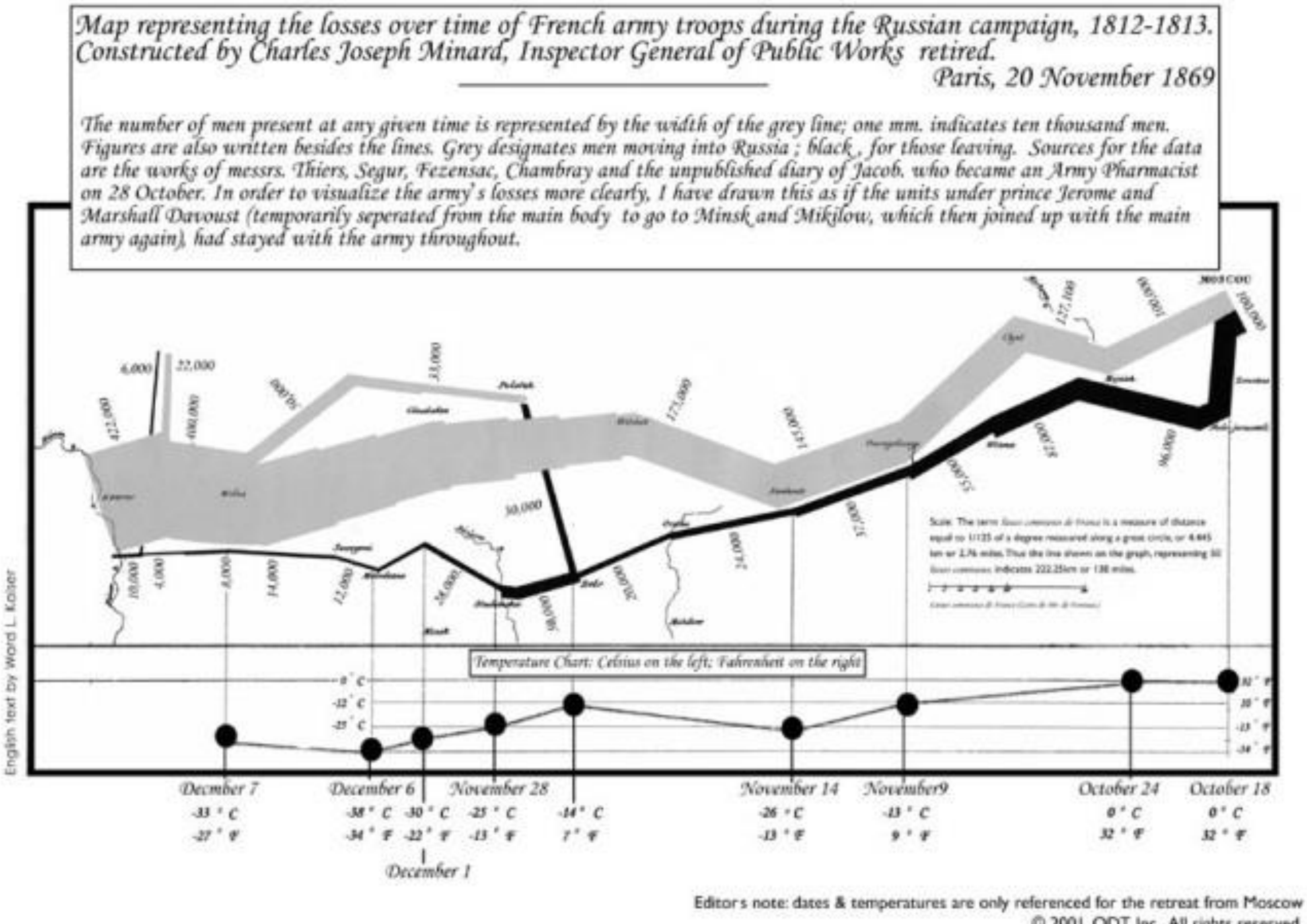


Figure 58. *Minard s map of Napoleon s Russian campaign.*
This graphic has been translated from French to English and modified to most effectively display the temperature data.

Invention of the Second Law of Thermodynamics by Sadi Carnot

$TdS = dE + PdV$

is arguably the most important equation in Science

If we were asked to predict what currently accepted principle would be valid 1,000 years from now,

The Second Law would be a good bet (Sean Carroll)

From this we can derive entropic forces **F** = T **grad** S

The S-B radiation law (const. T^4)

Information theory (Shannon, Gabor)

Accelerated expansion of the Universe

Even Gravity!

And much more modestly----

The design of thermodynamically efficient optics

The second law of Thermodynamics:

The invention of ENTROPY at the time of Carnot (1824), folks considered heat as a well-defined function $Q(T,V)$ for example and called it "Caloric" which was conserved but it's not. dQ is an example of a non-perfect differential,

Take $dQ = AdT + BdV$ which is not a perfect differential. If it were, then $\frac{\partial^2 Q}{\partial T \partial V} = \frac{\partial A}{\partial V} = \frac{\partial B}{\partial T}$ but this needs not be true. This can be extended to more than two variables.

Let's generalize $dQ = Xdx + Ydy + Zdz$ where X, Y, Z are functions of x, y, z and x, y, z can be temperature, volume etc.

Notice, if dQ were a perfect differential, say $d\sigma$, then $X = \frac{\partial \sigma}{\partial x}, Y = \frac{\partial \sigma}{\partial y}, Z = \frac{\partial \sigma}{\partial z}$ $and \frac{\partial Y}{\partial z} = \frac{\partial Z}{\partial y}, \frac{\partial Z}{\partial x} = \frac{\partial X}{\partial z}$ need not be true(these are called integrability conditions).

But sometimes, there exists an integrating factor, say $\tau(x, y, z)$ so $\frac{dQ}{\tau} = d\sigma = \frac{\partial \sigma}{\partial x} dx + \frac{\partial \sigma}{\partial y} dy + \frac{\partial \sigma}{\partial z} dz$. The point is, for Q, there is such a factor and it's called temperature. Due to Carrathodory 100 years ago! Math.Ann.67, 355(1909) and $\frac{dQ}{T} = dS$,S is the entropy or $TdS = dQ$.

The proof is based on the impossibility of certain conditions, for example heat flows from a cold body to a warm body(free cooling) dQ is 0 but I cool while you warm up. Other impossible conditions are converting heat to work with 100% efficiency and so on. ($dQ = 0$ is called adiabatic).

The proof goes like this: we know dQ is not a perfect differential. The adiabatic equation: $dQ = 0 = Xdx + Ydy + Zdz$, the points near (x, y, z) cannot fill a volume, if they did, all neighboring points are accessible, they don't just lie on a curve because $Xdx + Ydy + Zdz = 0$ is a tangent plane, so connecting the tangent plane that is a surface $\sigma(x, y, z)$ $such\ the\ \sigma =\ \ const$, passes through point (x, y, z).

Then since $dQ = 0$ confines points to the surface $d\sigma(x, y, z) = 0$

$$dQ = \tau(x, y, z) d\sigma(x, y, z)$$

And τ is called $integrating\ factor$.

For thermodynamics τ is the temperature T and σ is the entropy S

$$dQ = T(x, y, z) dS$$

make them

$$T = \tau \frac{d\sigma}{dx} = \frac{X}{\frac{\partial S}{\partial x}} = \frac{Y}{\frac{\partial S}{\partial y}} = \frac{Z}{\frac{\partial S}{\partial z}}$$

The differential of the heat, dQ for an infinitesimal quasi-static change when divided by the absolute T is a perfect differential dS of the entropy function.

Example, connect them to information by Shannon's theorem

$$S = -k_B \sum_n P_n \log P_n$$

Example, consider a string of bit 0, 1 such a string can carry information, say 1,0,0,0,1,1 etc. can represent all the letters of any alphabet hence any book content etc. Suppose we know what each bit is, the $P_n = 1, \log P_n = 0 \; and \; S = 0$. Perfect knowledge

Suppose we are clueless- Like tossing a coin, $P_n = 1/2$ all n and

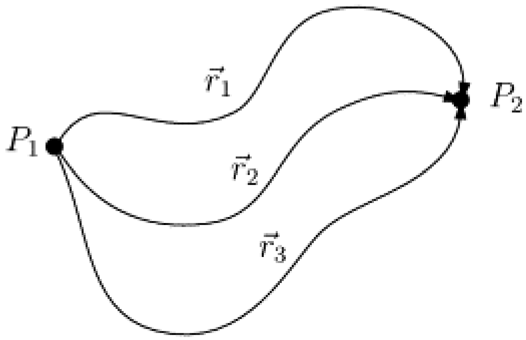
$$S = -k_B \sum_n \frac{1}{2} \log \frac{1}{2} = k_B \frac{\log 2}{2} \; N = k_B 0.346N$$

$$N = number \; of \; bits, k_B = Boltzmann's \; constant \approx 1.38 \times 10^{-23} (MKS \; units)$$

$$d^2s = d^2x + d^2y + d^2z$$
$$d^2s = (d^2\dot{x} + d^2\dot{y} + 1)d^2z$$

Define $\dot{x} = dx/dz$,$\dot{y} = dy/dz$

According to Fermat Principle $\delta\int\ n\,ds = 0$



$$\delta\int_{P_1}^{P_2} n\sqrt{(d^2\dot{x} + d^2\dot{y} + 1)}dz = 0$$

Define Lagrangian $L = n(x, y, z)\sqrt{(d^2\dot{x} + d^2\dot{y} + 1)}$

Then $\frac{\partial L}{\partial \dot{x}} = \frac{n\dot{x}}{\sqrt{(d^2\dot{x}+d^2\dot{y}+1)}} = p_x$

Then $\frac{\partial L}{\partial \dot{y}} = \frac{n\dot{y}}{\sqrt{(d^2\dot{x}+d^2\dot{y}+1)}} = p_y$

For a system that conserves its Hamiltonian, similar to Lagrangian equation:

$$\frac{d}{dz}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0,$$
$$\dot{p_x} = \frac{\partial L}{\partial x}$$

Therefore

$$\frac{\partial L}{\partial \dot{x}} = p_x, \frac{\partial L}{\partial \dot{y}} = p_y, \dot{p_x} = \frac{\partial L}{\partial x}, \dot{p_y} = \frac{\partial L}{\partial y}$$

The Hamiltonian of the system is:

$$H = p_x\dot{x} + p_y\dot{y} - L(x, y, \dot{x}, \dot{y})$$

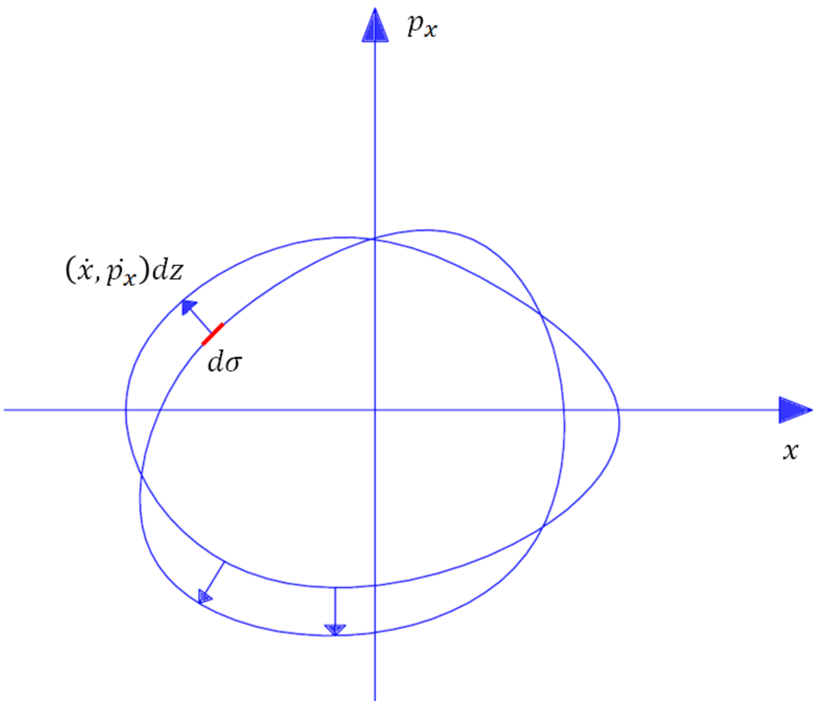
$$dH = p_x\, d\dot{x} + dp_x\, \dot{x} + p_y d\dot{y} + dp_y \dot{y} - \left(\frac{\partial L}{\partial x}dx + \frac{\partial L}{\partial \dot{x}}d\dot{x} + \frac{\partial L}{\partial y}dy + \frac{\partial L}{\partial \dot{y}}d\dot{y}\right)$$

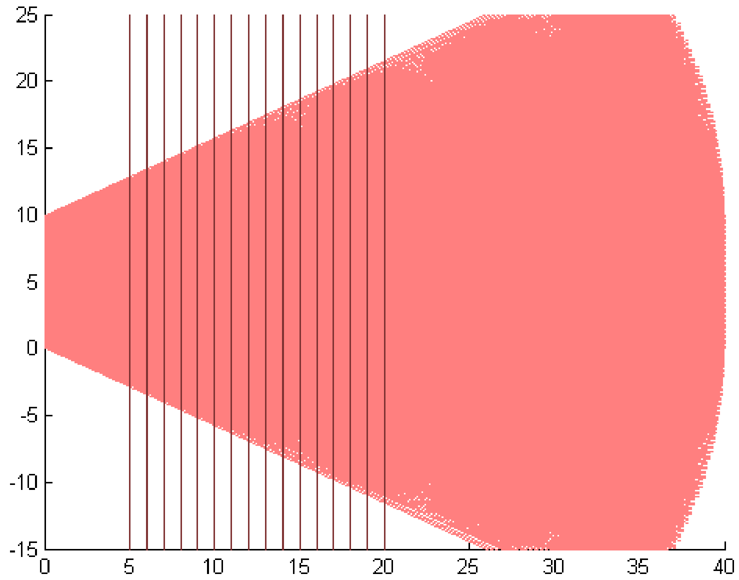
$$= -\dot{p}_x dx + \dot{x} dp_x - \dot{p}_y dy + \dot{y} dp_y$$

$$\frac{\partial H}{\partial x} = -\dot{p}_x, \frac{\partial H}{\partial y} = -\dot{p}_y, \frac{\partial H}{\partial p_x} = \dot{x}, \frac{\partial H}{\partial p_y} = \dot{y}$$

$$\frac{\partial \dot{p}_x}{\partial p_x} = -\frac{\partial H}{\partial p_x}\left(\frac{\partial H}{\partial x}\right) = \frac{\partial H}{\partial x}\left(\frac{\partial H}{\partial p_x}\right) = \frac{\partial \dot{x}}{\partial x}$$

Define a four vector field $W = (\dot{x}, \dot{p}_x, \dot{y}, \dot{p}_y)$ then the 4 dimensional divergence of W is 0. If we use W on a closed surface $\sigma(x, P_x, y, P_y)$ of a four space $v(x, P_x, y, P_y)$, then the incremental volume along dz covered by such a surface is $dV = \int_\sigma W dz d\sigma$ According to Gauss's theorem, $\int_\sigma W d\sigma = \int_v Div\ W\ dv = 0$, therefore the enclosed volume $dV = 0. \int_\sigma dxdydp_xdp_y = constant$





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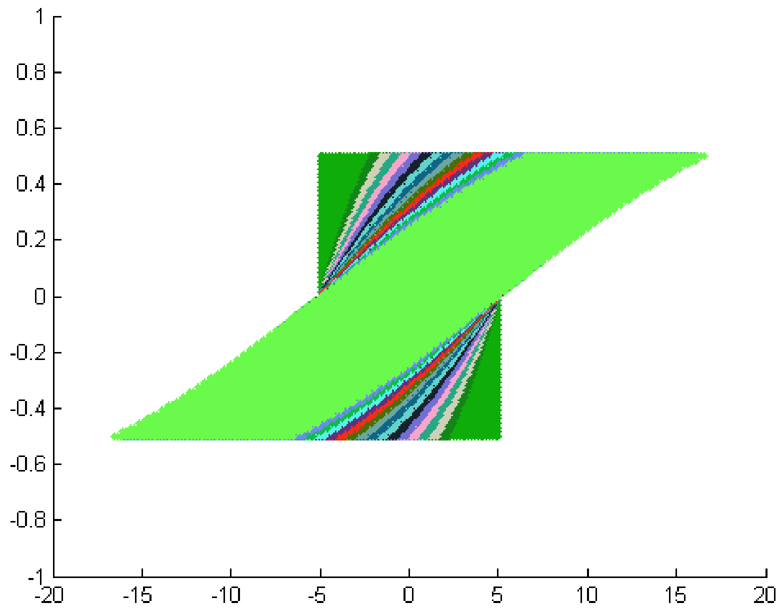


1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-20
-15
-10
-5
0
5
10
15
20

Limits to Concentration

- from λ $_{\text{max sun}} \sim$ 0.5 μ

  we measure $T_{\text{sun}} \sim$ 6000° (5670°)

  Without actually going to the Sun!

- Then from σT^4 - solar surface flux~ 58.6 W/mm^2
  - The solar constant ~ 1.35 mW/mm^2
  - The second law of thermodynamics
  - C max ~ 44,000
  - Coincidentally, C max = $1/\sin^2\theta$
  - This is evidence of a deep connection to optics

Imaging Devices and Their Limitations

- If one were to ask the proverbial "man on the street" for a suggestion of how one might attain the highest possible level of concentration of, say solar flux a plausible response would be to use a good astronomical telescope, perhaps the 200 inch telescope on Mt. Palomar, or whatever one's favorite telescope might be.

  * Of course such an experiment had better remain in the realm of imagination only, since beginning astronomers are admonished never to point their telescope at the sun or risk catastrophic consequences to the instrument.

- But …

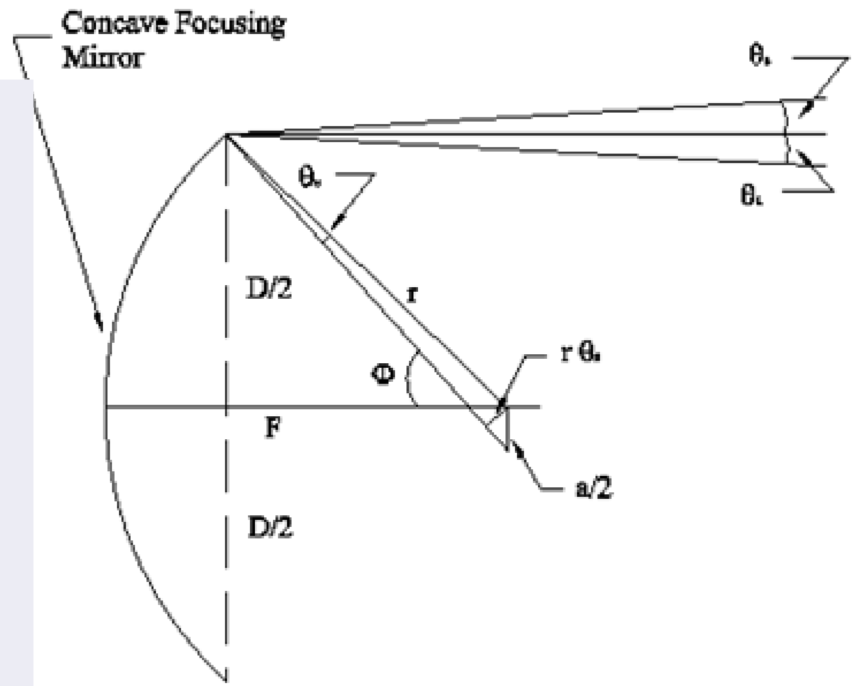
Concentration Limit of a Telescope

- Concentration limit

  $$\sin^2 2\phi / 4\sin^2\theta$$

  – ϕ: rim angle of the telescope
- Best concentration achieved

  $1/4\sin^2\theta$ when $\phi = 45^0$
- Falls short of the fundamental limit by a factor 4!
  – Now factors of 4 are significant in technology (and many other forms of human endeavor)



$D = 2\,r \sin\phi \qquad C = (D/d)^2 = (1/4)\sin^2 2\phi / \sin^2\theta_s$

$\leq (1/4)1/\sin^2\theta_s \leq \underline{(1/4)C_{max}}$

$d = 2\,r \sin\theta / \cos\phi$

$D/d = \sin\phi\cos\phi / \sin\theta$

$= \sin 2\phi / 2\sin\theta$

Concentration Limit in 2-D Cases

- Entirely similar considerations can be applied to 2-D or trough concentrators.

- A straightforward generalization to a strip absorber rather than a disk absorber gives a limit for say, a parabolic trough of $\sin 2\phi / 2\sin\theta$

- Upper limit: $1//2\sin\theta$, for rim angle $\phi = 45^0$.
  - This would be a useful configuration for a photo-voltaic concentrator, with the strip consisting of solar cells.

•A more useful geometry for a parabolic trough thermal concentrator is a tubular receiver.

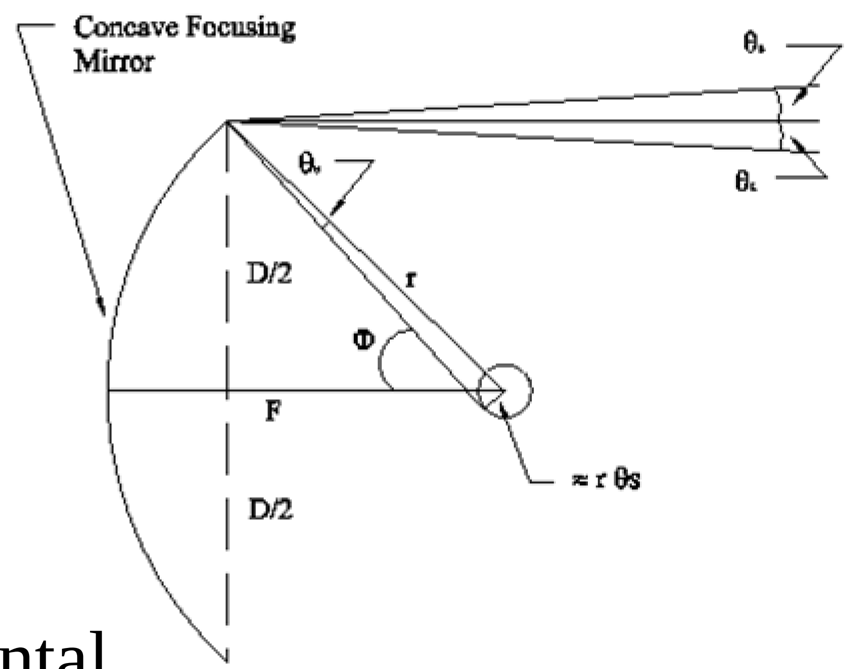
- Concentration relation

  $\sin\phi / \pi \sin\theta$

- Maximum value

  $1/\pi \sin\theta$ at 90^0 rim angle.

- Falls short of the fundamental limit by a factor π !



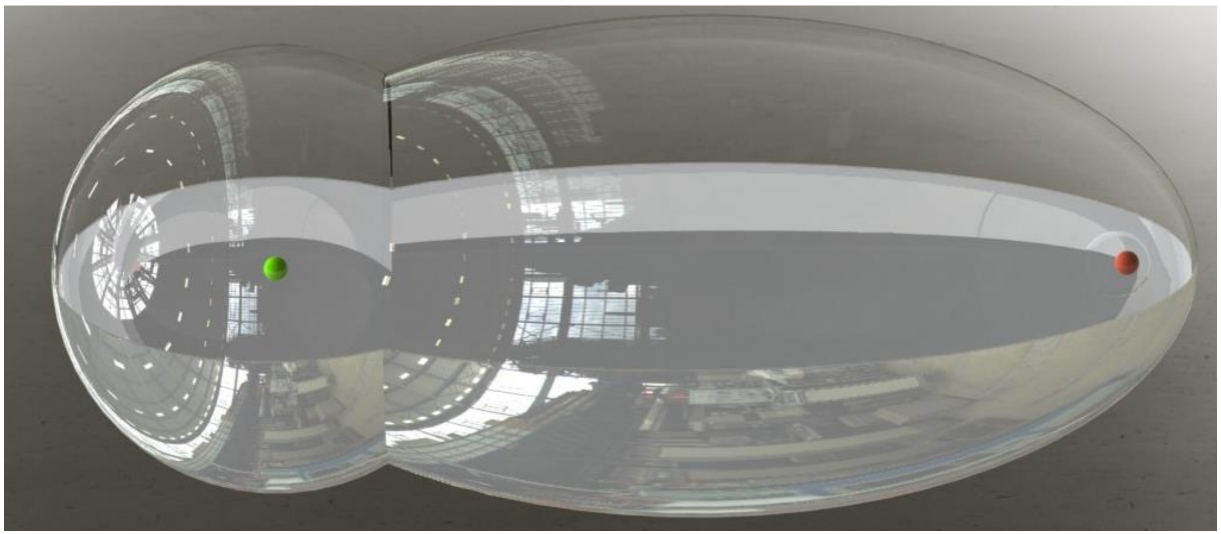
$$C = D/2\pi r \sin\theta_s = \sin\phi / \pi \sin\theta_s$$
$$\leq 1/\pi \sin\theta_s \leq \underline{(1/\pi)\, C_{max}}$$

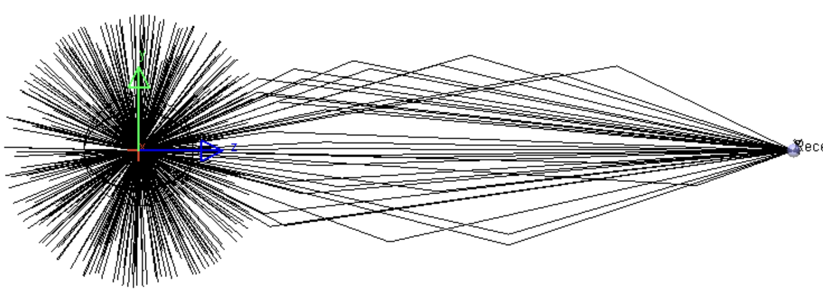
Nonimaging Concentrators

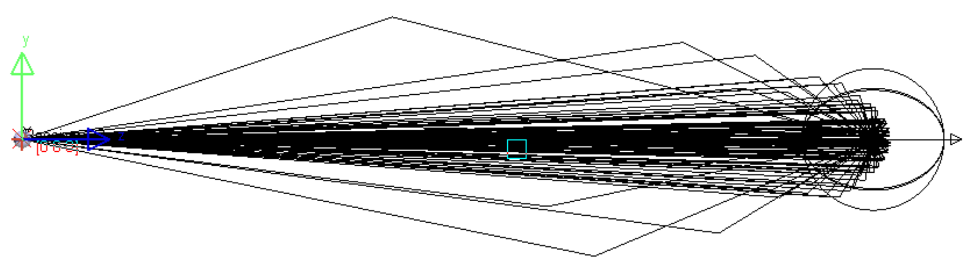
- It was the desire to bridge the gap between the levels of concentration achieved by common imaging devices, and the *sine law of concentration limit* that motivated the invention of nonimaging optics.

Failure of conventional optics

$F_{AB} \ll F_{BA}$ where F_{AB} is the probability of radiation starting at A reaching B--- etc







First and Second Law of Thermodynamics

Nonimaging Optics is the theory of maximal efficiency **radiative transfer**

It is axiomatic and algorithmic based

As such, the subject depends much more on thermodynamics than on optics

To learn efficient optical design, first study the theory of furnaces.

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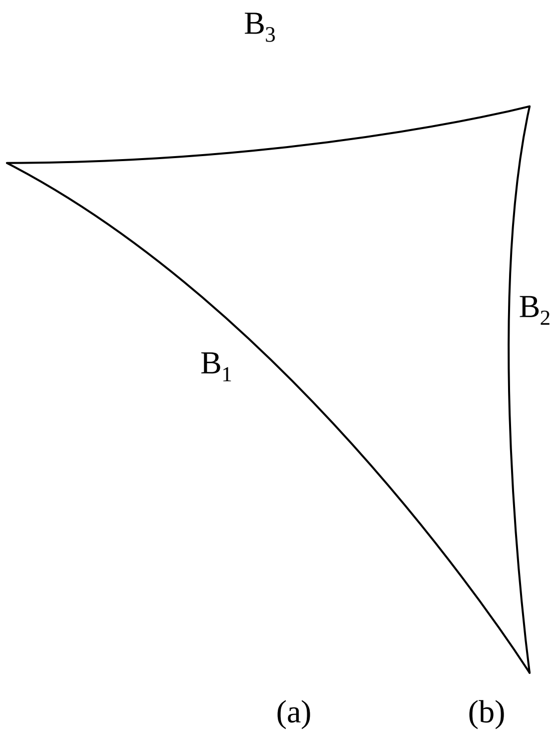
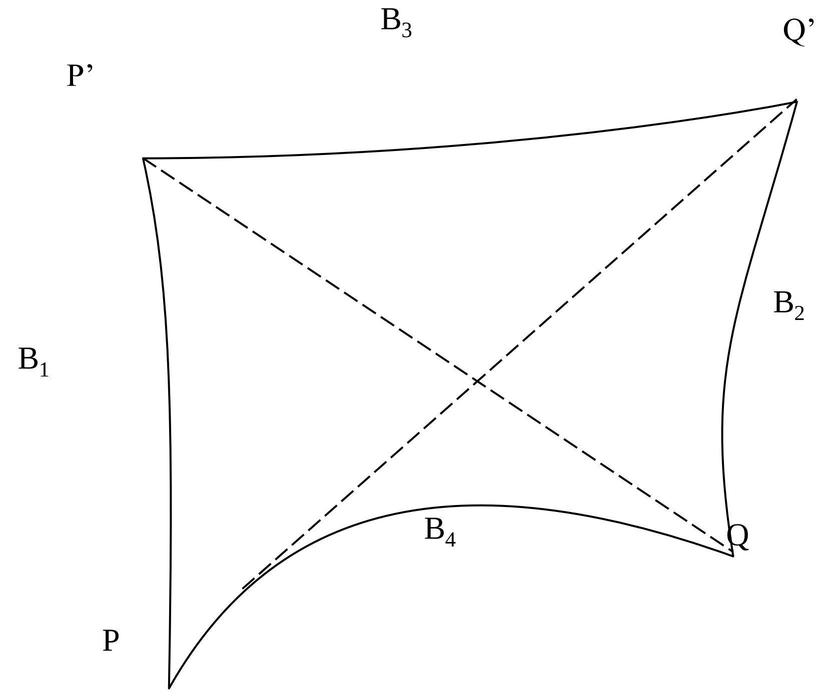




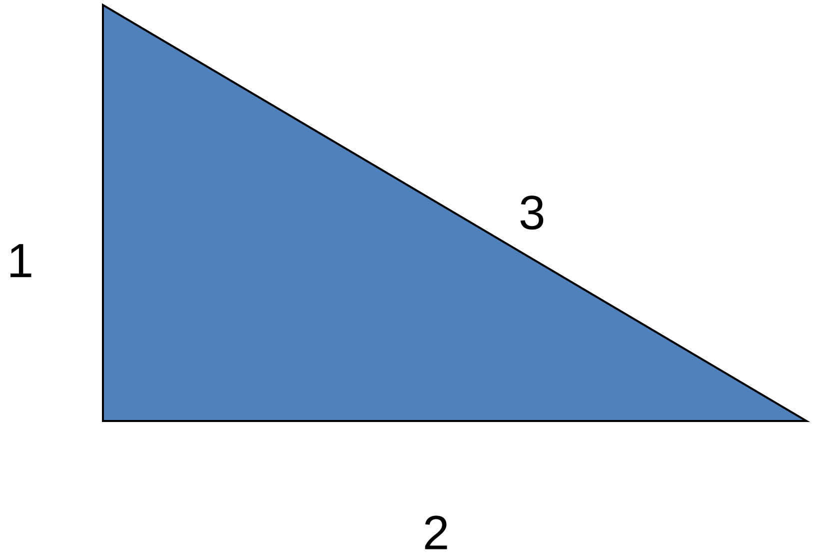
Radiative transfer between walls in an enclosure

HOTTEL STRINGS

Michael F. Modest, Radiative Heat Transfer, Academic Press 2003

Hoyt C. Hottel, 1954, Radiant-Heat Transmission, Chapter 4 in
William H. McAdams (ed.), *Heat Transmission,* 3rd ed. McGRAW-HILL

Strings 3-walls



$P12 = (A1 + A2 – A3)/(2A1)$

$P13 = (A1 + A3 – A2)/(2A1)$

$P23 = (A2 + A3 – A1)/(2A2)$

$qij = AiPij$

$Pii = 0$

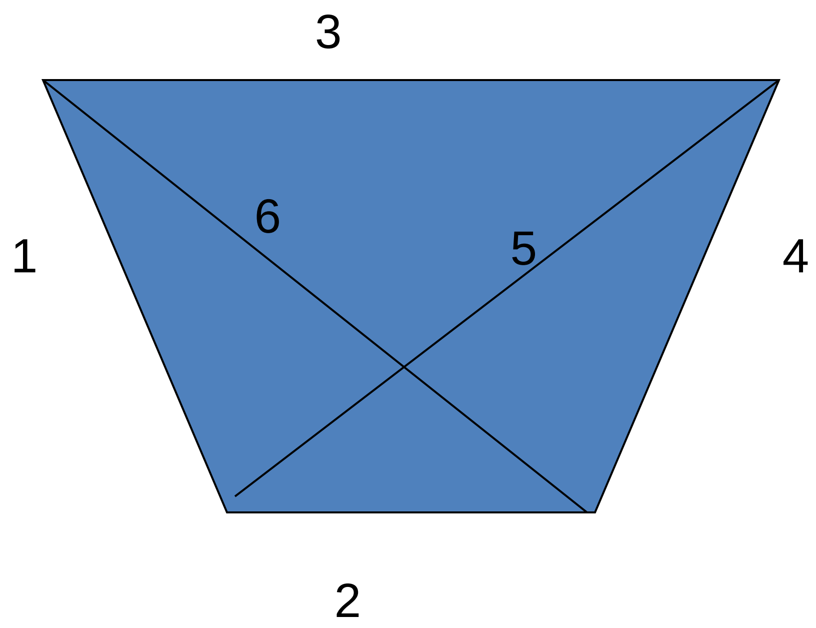
$P12 + P13 = 1$
$P21 + P23 = 1$
$P31 + P32 = 1$

3 Eqs

$Ai\ Pij = Aj\ Pji$

3 Eqs

Strings 4-walls



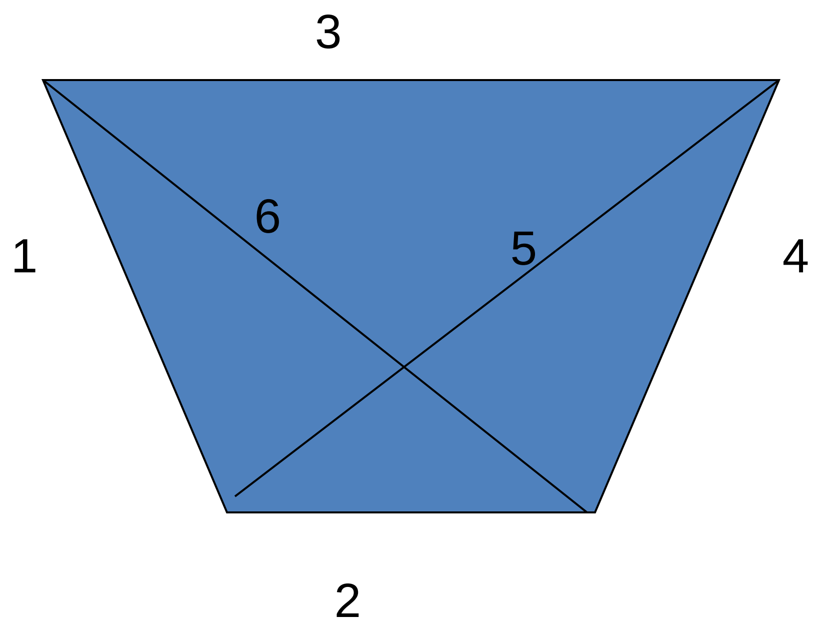
P12 + P13 + P14 = 1

P21 + P23 + P24 = 1

P14 = [(A5 + A6) – (A2 + A3)]/(2A1)

P23 = [(A5 + A6) – (A1 + A4)]/(2A2)

Limit to Concentration



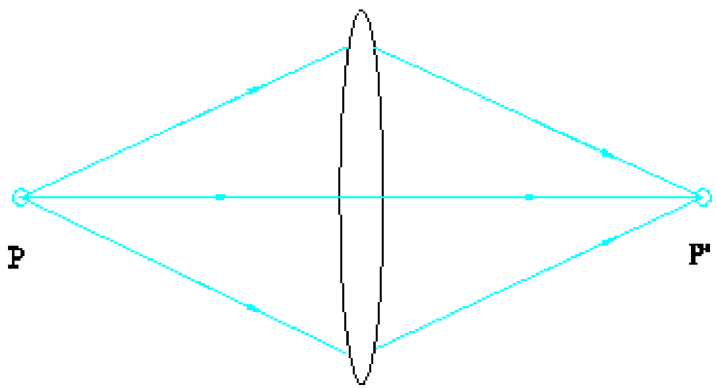
P23 = [(A5 + A6) – (A1 + A4)]/(2A2)S

P23= sin(θ) as A3 goes to infinity

- This rotates for symmetric systems to sin $^2(\theta)$

Fermat's Principle for Rays and Strings

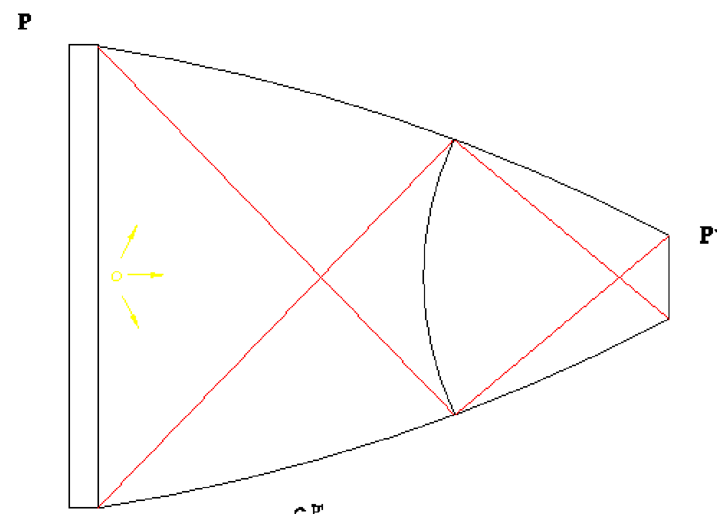
Imaging Optics



$$\int_{P\ \text{ray}}^{P'} n\, dl = \text{constant} \quad \text{[Fermat 1601-1665]}$$, where

n = index of refraction
l = path length

Nonimaging Optics:



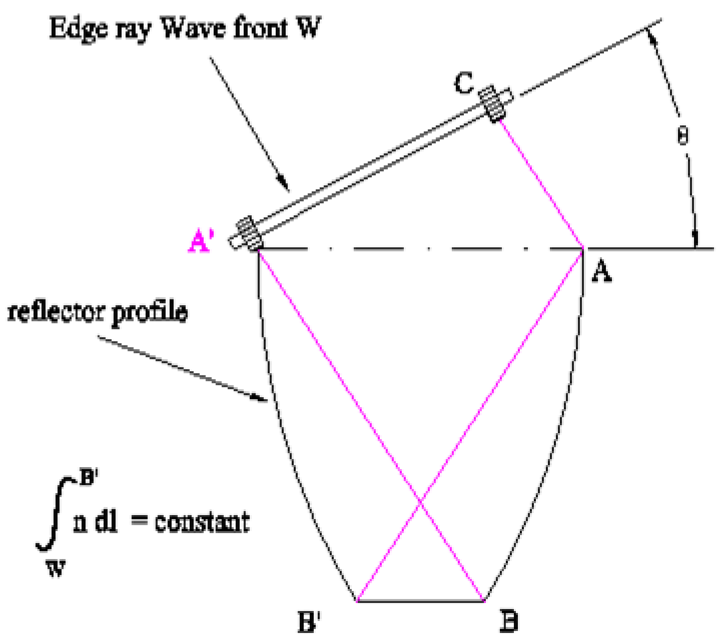
$$\int_{P\ \text{string}}^{P'} n\, dl = \text{constant}$$

- What are strings?

String Method

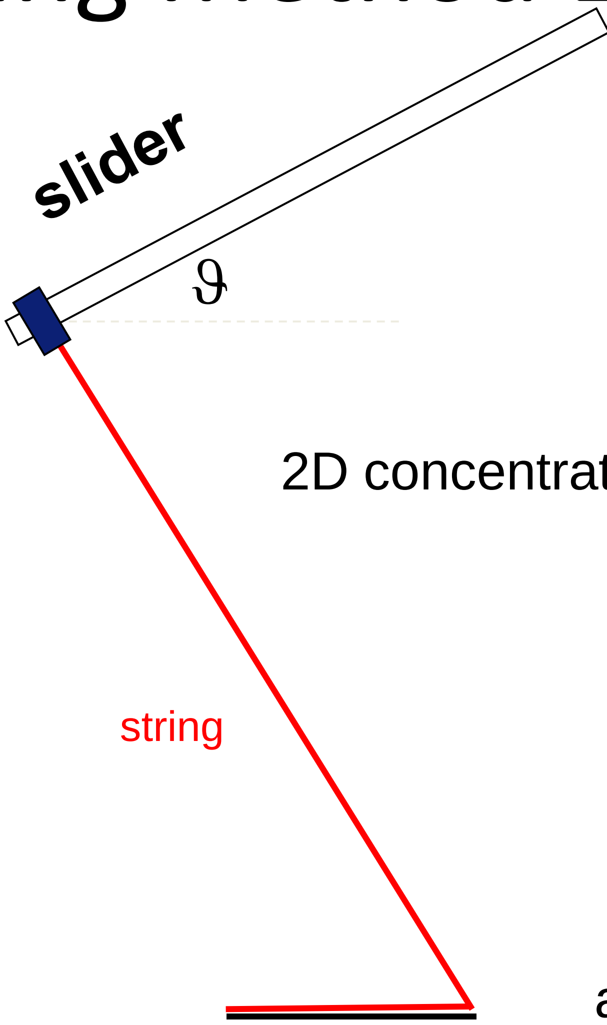
- We explain what strings are by way of example.
- We will proceed to solve the problem of attaining the sine law limit of concentration for the simplest case, that of a flat absorber.

String Method Example: CPC



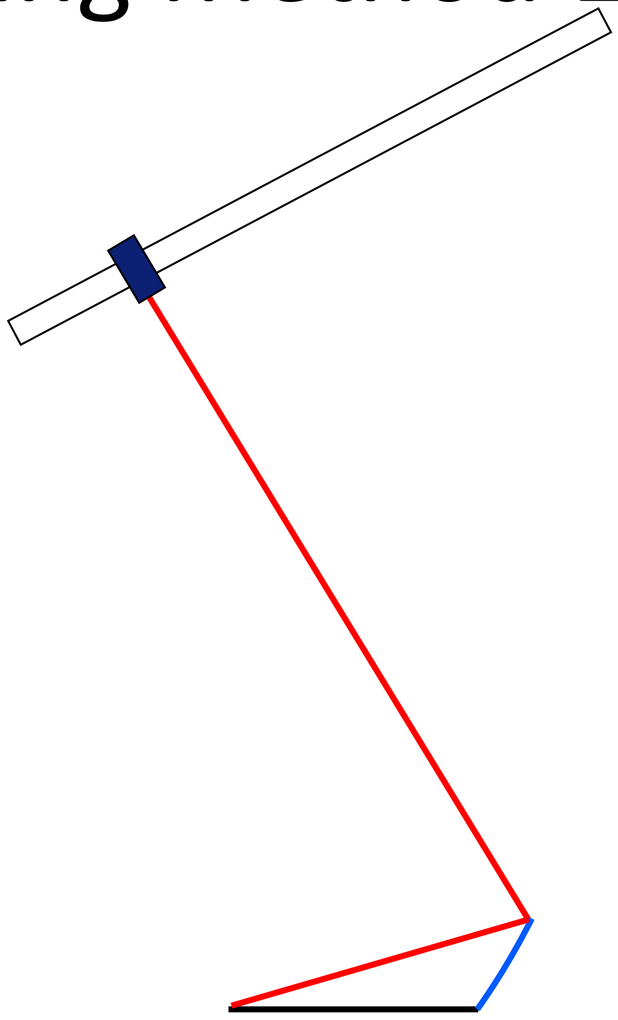
- We loop one end of a “string” to a “rod” tilted at angle θ to the aperture AA’ and tie the other end to the edge of the exit aperture B’.
- Holding the length fixed, we trace out a reflector profile as the string moves from C to A’.

String Method Example: CPC

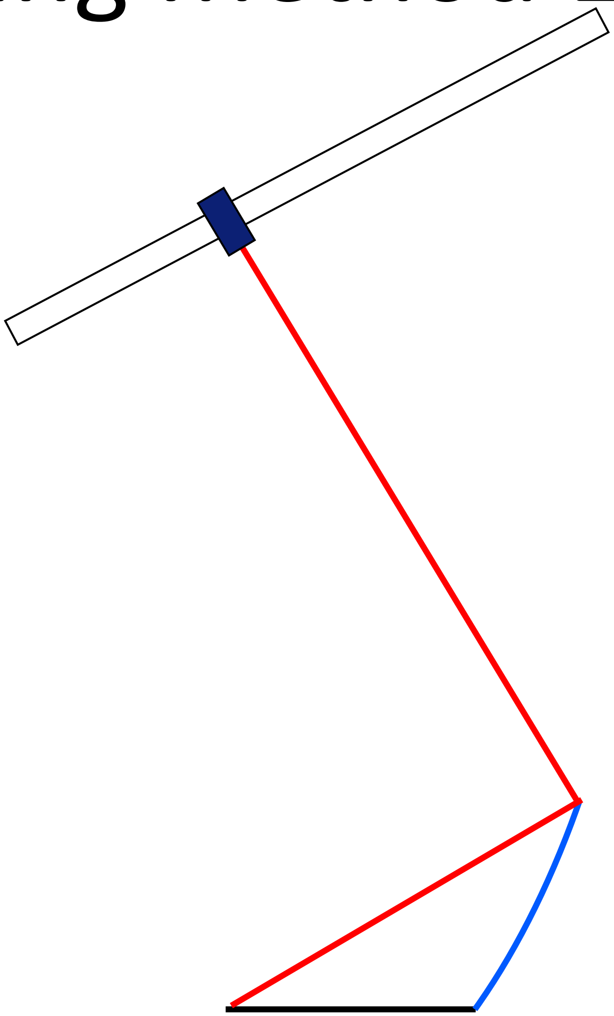


2D concentrator with acceptance (half) angle ϑ

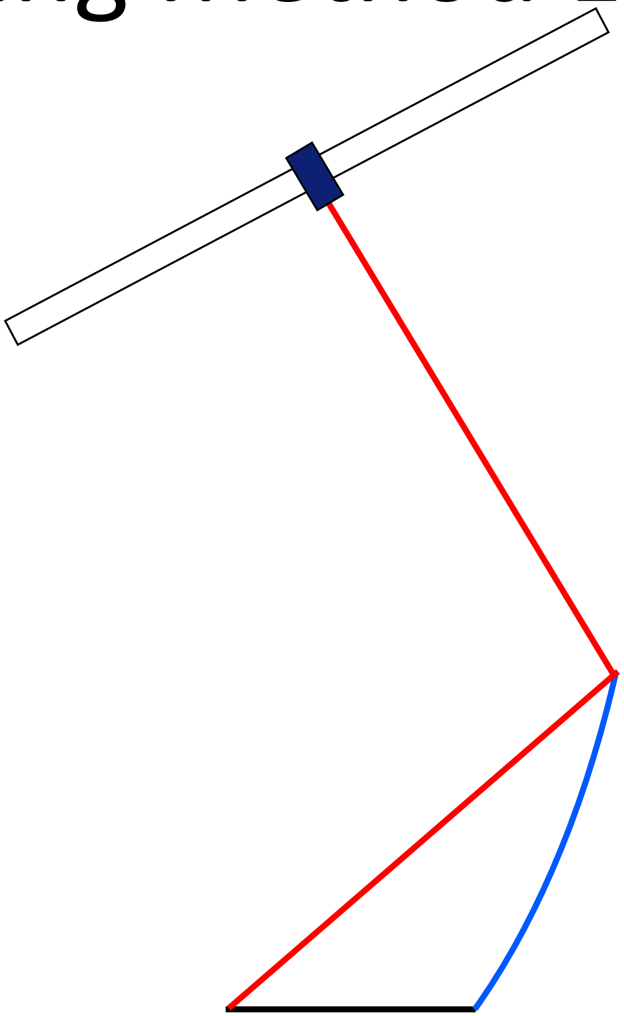
String Method Example: CPC



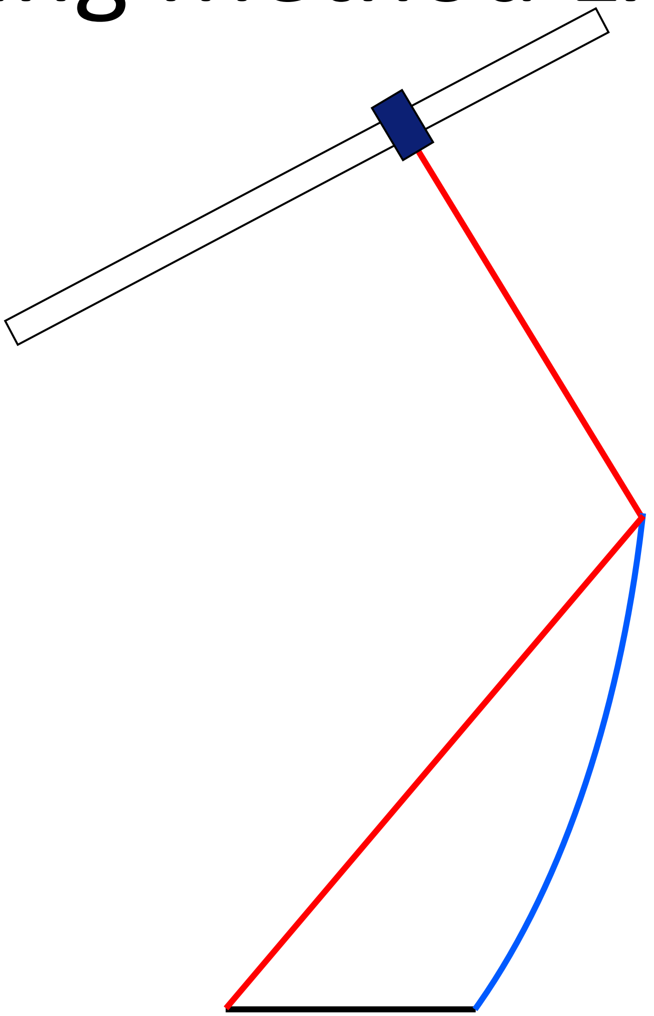
String Method Example: CPC



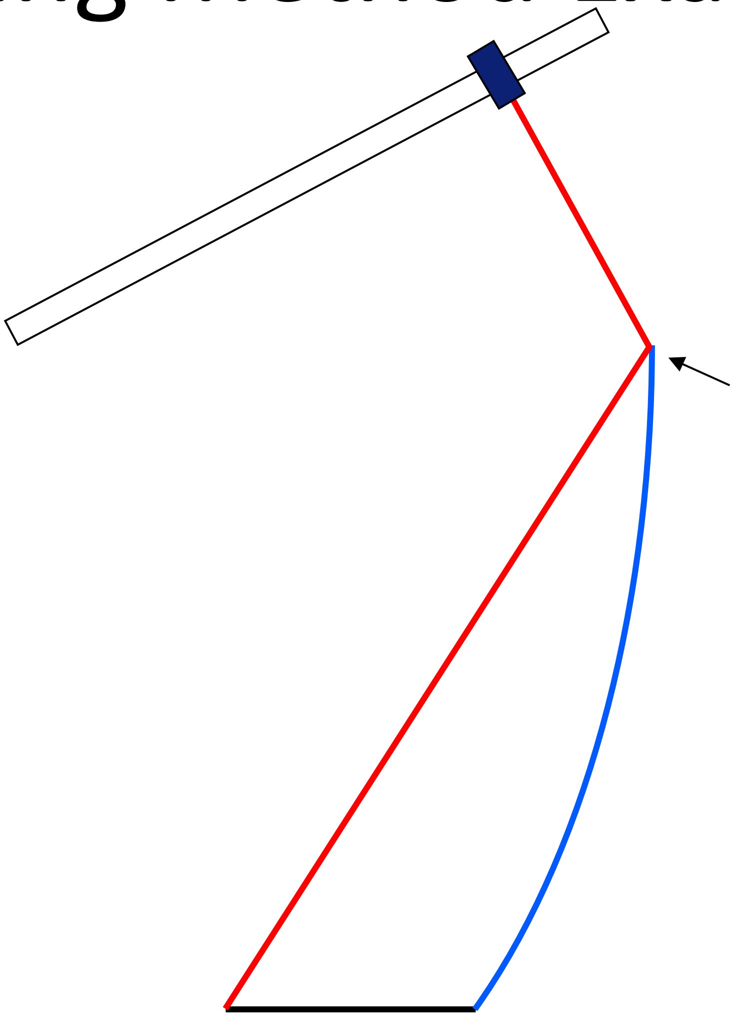
String Method Example: CPC



String Method Example: CPC

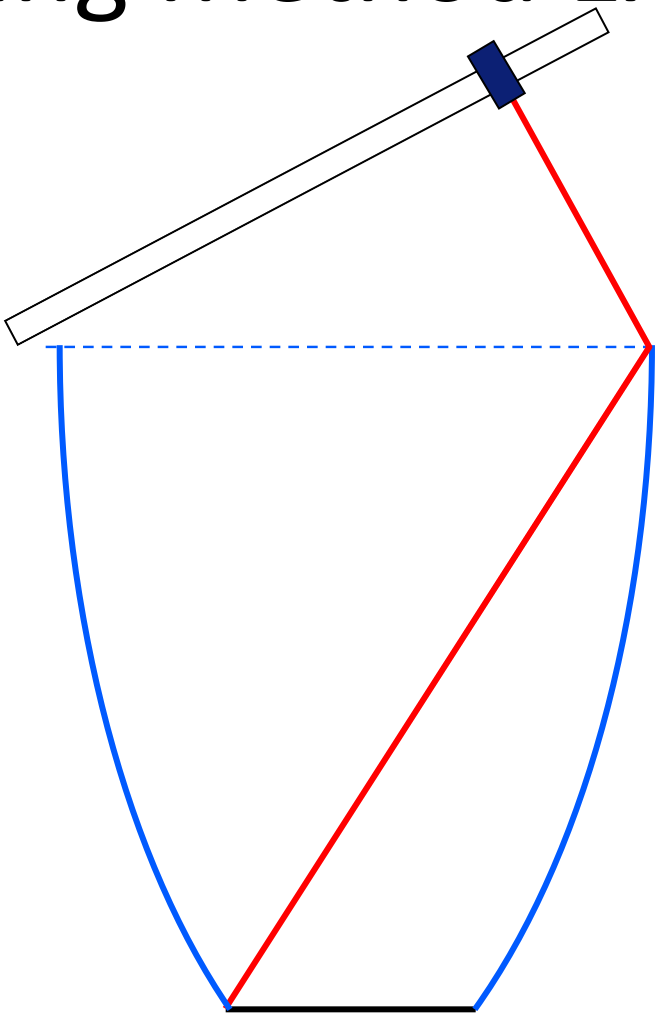


String Method Example: CPC

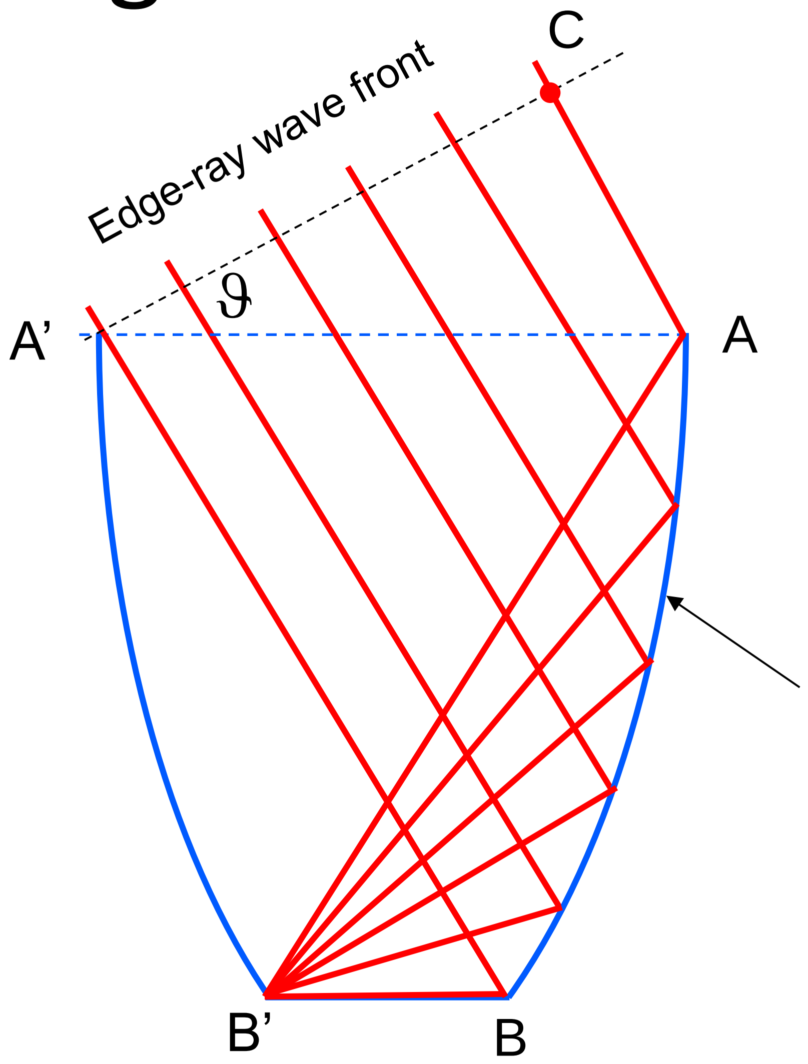


stop here, because slope becomes infinite

String Method Example: CPC



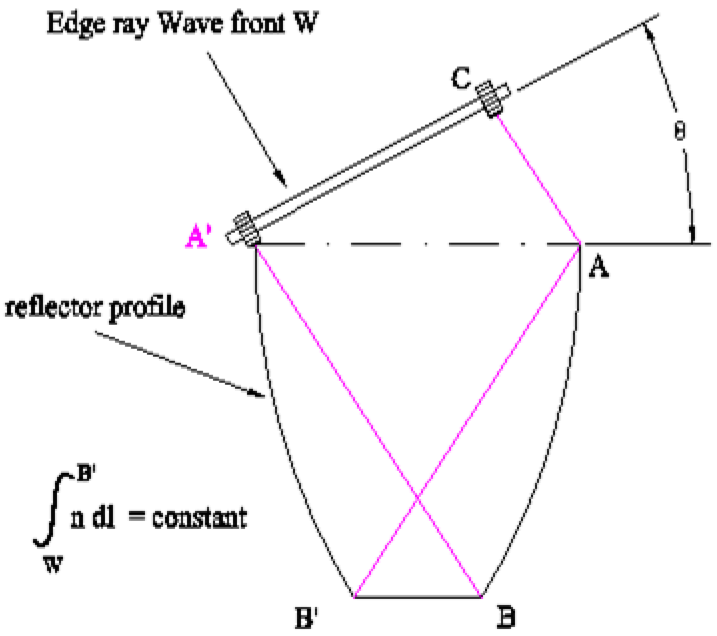
String Method Example: CPC



Compound Parabolic Concentrator (CPC)

(tilted parabola sections)

String Method Example: CPC



$$B'A + AC = B'B + BA'$$

$$B'A = BA'$$

$$BB' = AC = AA'\sin\vartheta$$

$$\Rightarrow AA'\sin\vartheta = BB'$$

$$\boxed{\begin{array}{l} C = \dfrac{AA'}{BB'} = \dfrac{1}{\sin\vartheta} \\[2ex] C(\text{cone}) = \left(\dfrac{AA'}{BB'}\right)^2 = \dfrac{1}{\sin^2\vartheta} \end{array}}$$

sine law of concentration limit!

String Method Example: CPC

- The 2-D CPC is an ideal concentrator, i.e., it works perfectly for all rays within the acceptance angle θ,
- Rotating the profile about the axis of symmetry gives the 3-D CPC
- The 3-D CPC is very close to ideal.

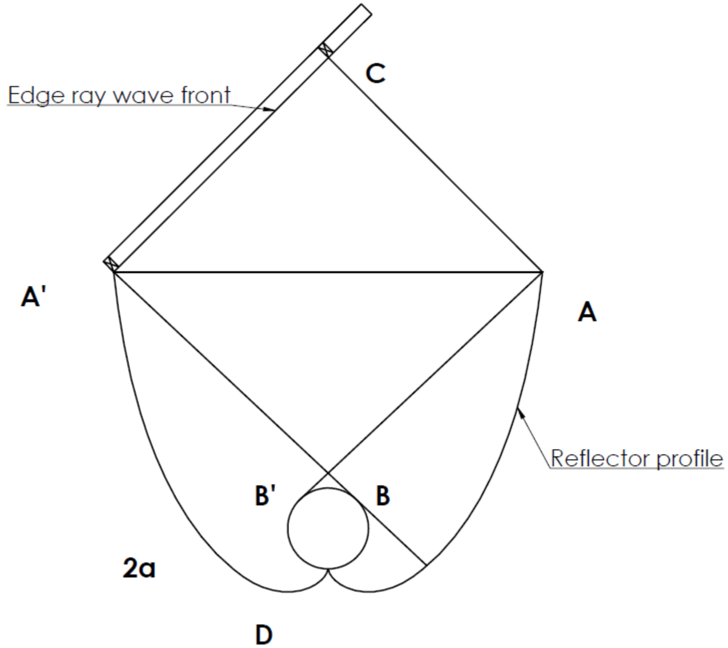
String Method Example: CPC

- Notice that we have kept the *optical length* of the string fixed.
- For media with varying index of refraction (n), the physical length is multiplied by n.

- The string construction is very versatile and can be applied to *any* convex (or at least non-concave) absorber…

I am frequently asked- Can this possibly work?

String Method Example: Tubular Absorber



String method: $\int_{w}^{D} ndl = Constant$

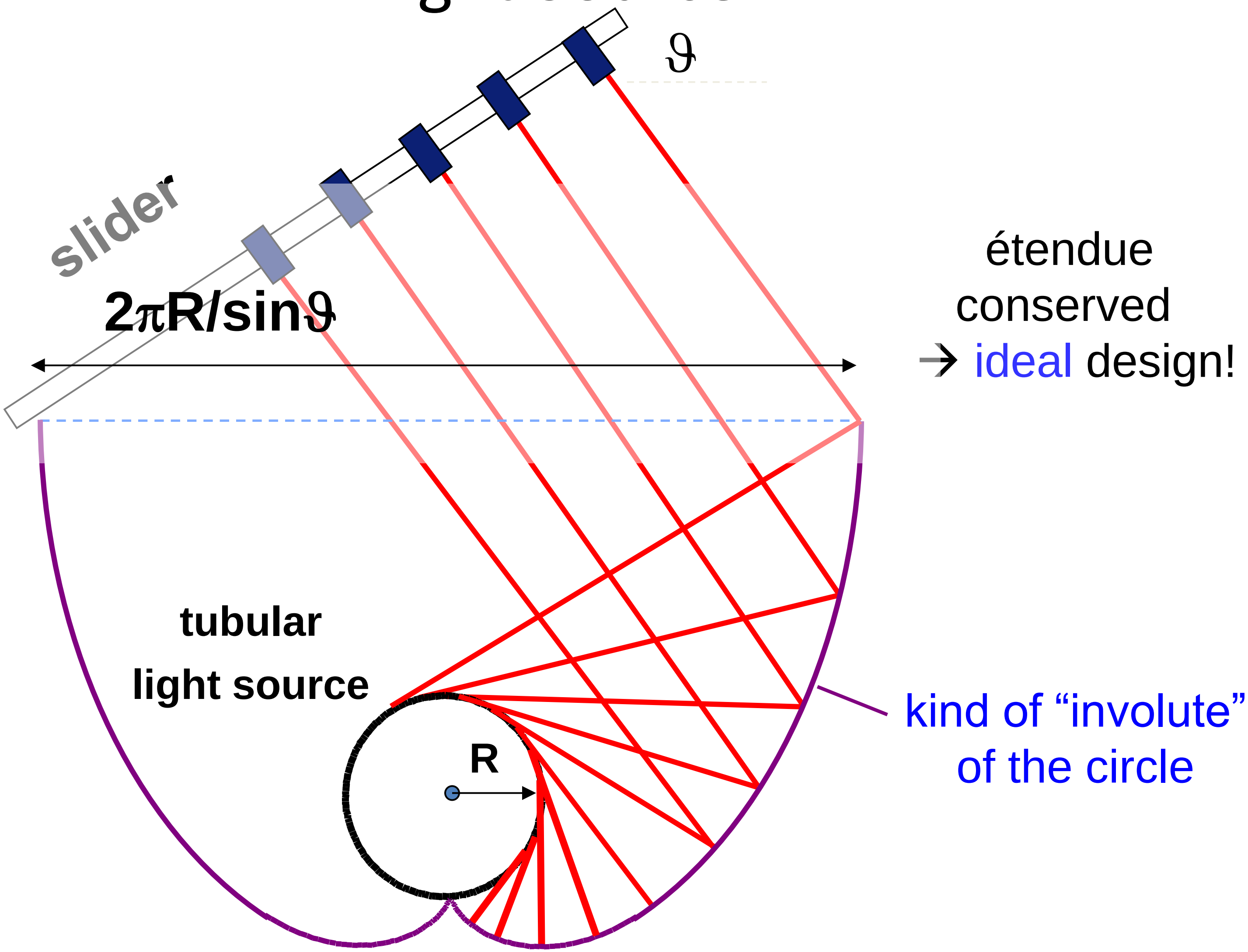
AC+AB'+B'D=A'B+BD+2πa

AA' sinθ=2πa

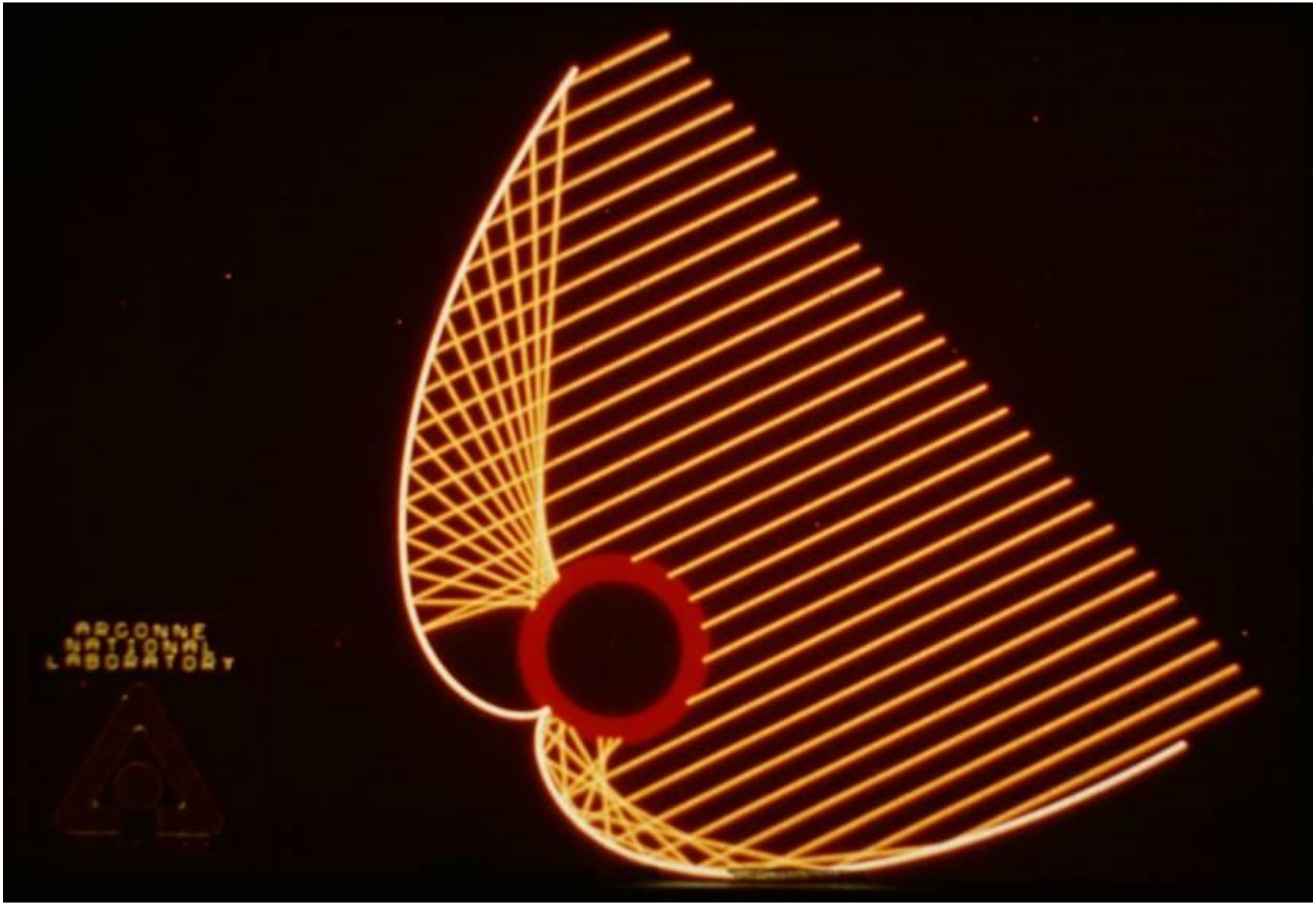
$$C = \frac{AA'}{2\pi a} = \frac{1}{\sin \vartheta}$$

- String construction for a tubular absorber as would be appropriate for a solar thermal concentrator.

String Method Example: Collimator for a Tubular Light Source



Non-imaging Concentrator



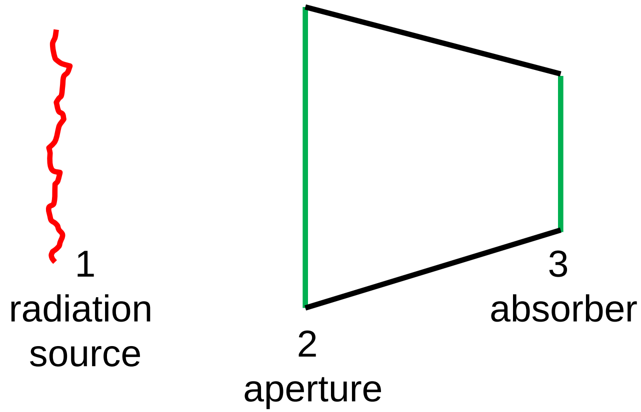
The connection between entropy and information is well known.[17,18] The entropy of a system measures one's uncertainty or lack of information about the actual internal configuration of the system. Suppose that all that is known about the internal configuration of a system is that it may be found in any of a number of states with probability p_n for the nth state. Then the entropy associated with the system is given by Shannon's formula[17,18]

$$S = -\sum_n p_n \ln p_n \quad . \tag{10}$$

The conventional unit of information is the "bit" which may be defined as the information available when the answer to a yes-or-no question is precisely known (zero entropy). According to the scheme (11) a bit is also numerically equal to the maximum entropy that can be associated with a yes-or-no question, i.e., the entropy when no information whatsoever is available about the answer. One easily finds that the entropy function (10) is maximized when $p_{yes} = p_{no} = \frac{1}{2}$. Thus, in our units, one bit is equal to ln2 of information.

S = k (½ log2) N

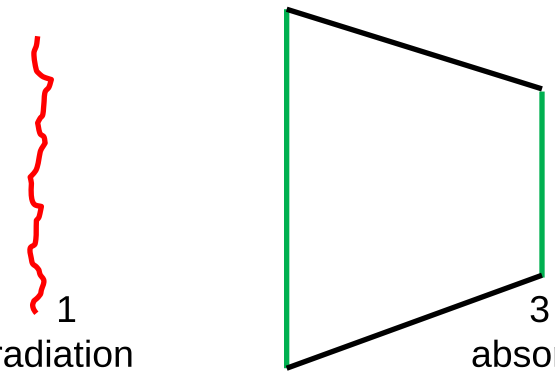
The general concentrator problem



Concentration C is defined as A2/A3

What is the "best" design?

Characteristics of an optimal concentrator design



Let Source be maintained at T_1 (sun)

Then T_3 will reach $T_1 \leftrightarrow P_{31} = 1$

Proof: $q_{13}=\sigma T_1^4 A_1 P_{13} = \sigma T_1^4 A_3 P_{31}$

But $q_{3total}=\sigma T_3^4 A_3 \geq q_{13}$ at steady state

$T_3 \leq T_1$ (second law)$\rightarrow P_{31}=1 \leftrightarrow T_3=T_1$

1st law efficiency: energy conservation

$$q_{12} = q_{13} \Rightarrow P_{12} = P_{13}$$

2nd law efficiency:

$$A_1 P_{12} = A_1 P_{13}, but\ A_1 P_{13} = A_3 P_{31}$$

The concentration ratio C:

$$C = \frac{A_2}{A_3}$$

$$A_3 = \frac{A_1 P_{12}}{P_{31}}$$

The maximum concentration ratio

C_{max} corresponds to minimum A_3

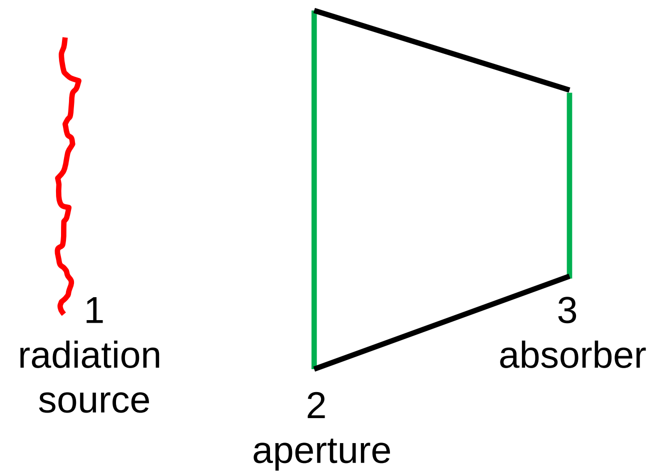
C is maximum IFF $P_{31} = 1$

Recall that for maximum thermodynamics efficiency

$$A_1 P_{12} = A_1 P_{13} = A_3$$

Then $A_2 P_{21} = A_3$

$$C_{max} = 1/P_{21}$$

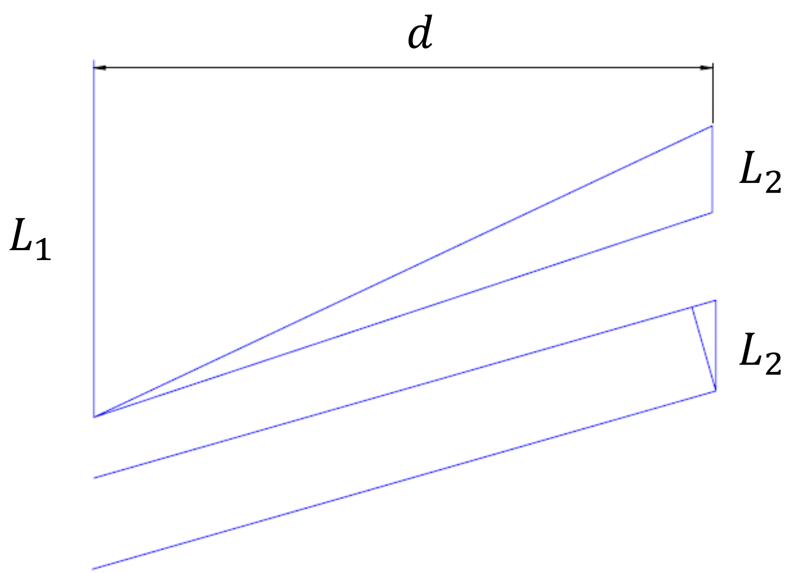


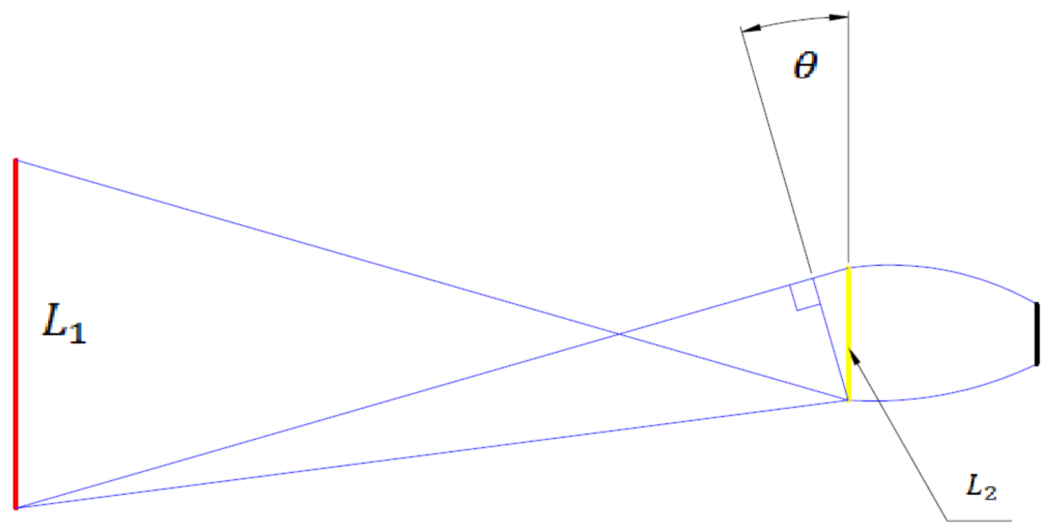
Hospital in Gurgaon, India DEC 2011

Roland, I hope Shanghai went well Hit 200C yesterday with just 330W DNI. Gary D. Conley~*Ancora Imparo*

www.b2uSolar.com

$$Long\ string - Short\ String = L_2 sin\theta$$
$$P_{21} = sin\theta$$





So that $C_{max} = 1/sin\theta$,
Notice θ is the maximum angle of radiation incident on A_2
Generalize to 3-dimensional, rotational symmetry.

$$P_{21} = \left(\frac{long\ string - short\ string}{L_2}\right)^2 = (sin\theta)^2$$

$$C_{max} = \left(\frac{1}{sin\theta}\right)^2$$

An alternative way to get the sine law is to consider the angular momentum with respect to the axis of symmetry.

$$\vec{J} = \vec{r} \times \vec{P}$$

$$\vec{P} = n(L, M, N)$$

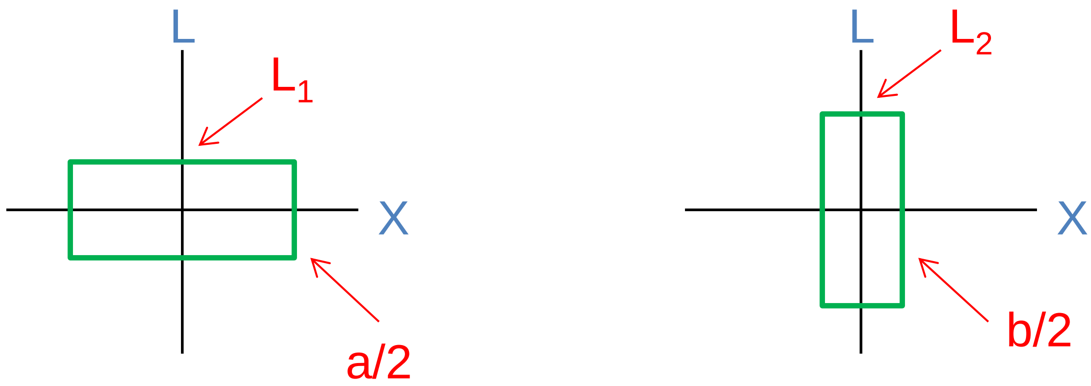
J_z is conserved

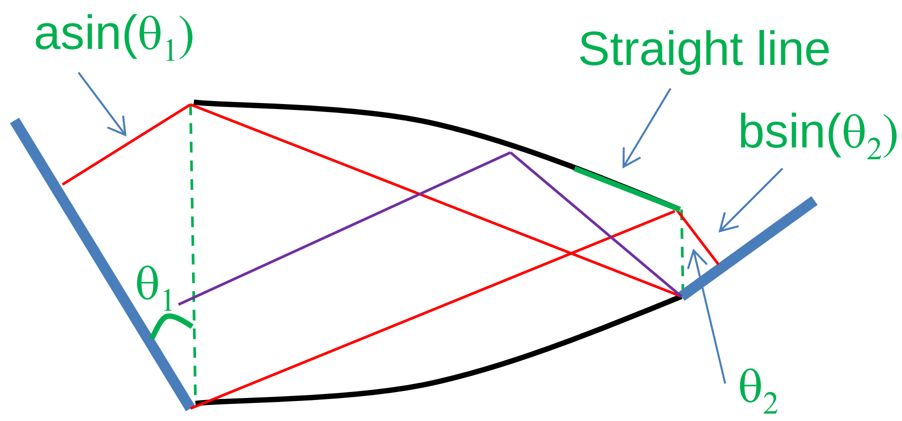
Designing the thermodynamically efficient concentrator

We have used "strings" to find the limit of concentration, if we can use the strings to design the optical system, we have a chance of meeting the limit.

θ_1/θ_2 angle transformer

Etendue conservation $L_1 a = L_2 b$, $L_1 = \sin\theta_1, L_2 = \sin\theta_2, a\sin\theta_1 = b\sin\theta_2$

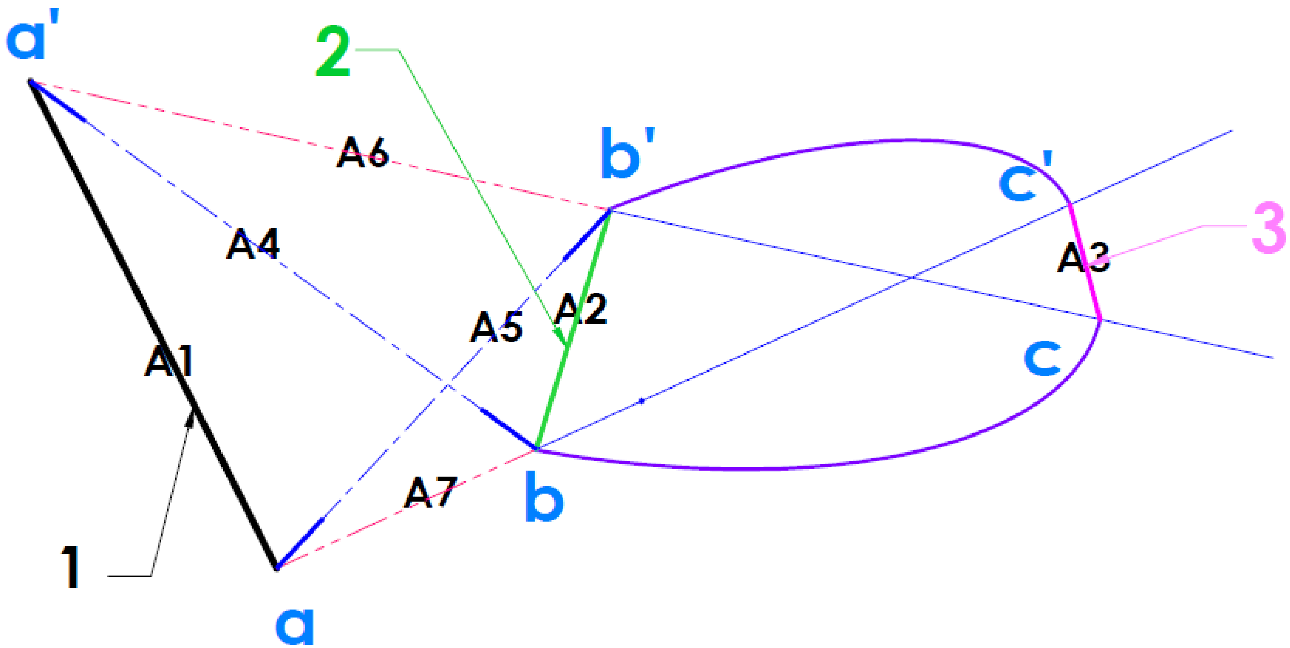




asin(θ1)
Straight line
bsin(θ2)
θ1
θ2

String method deconstructed

- 1. Choose source
- 2.Choose aperture
- 3. Draw strings
- 4. Work out $P_{12}A_1$



5. $P_{12}A_1 = \frac{1}{2}[\sum long\ strings - \sum short\ strings] = A_3 = 0.55A_1 = 0.12A_1$

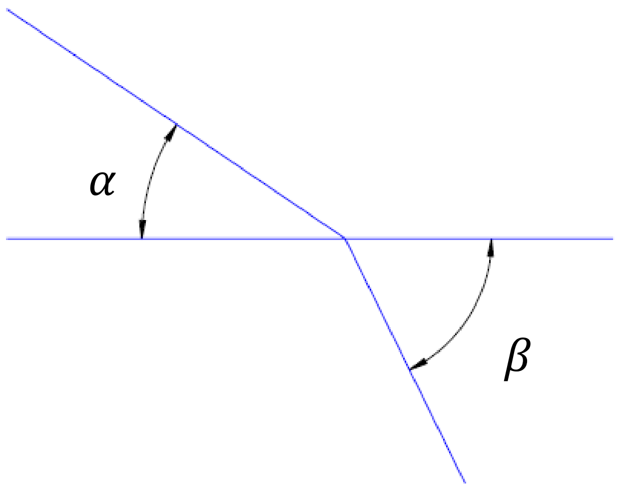
6. Fit A_3 between extends strings => 2 degrees of freedom, Note that

$$A_3 = cc' = \frac{1}{2}[(ab' + a'b - (ab + a'b')]$$

7. Connect the strings.

Index of Refraction (n)

Recall $C_{max} = n/\sin\theta$, so if $\theta = \frac{\pi}{2}$, we should be able to concentration by n (or n^2in three dimension). How? Consider the air/dielectric interface.



By momentum conservation,

$$\cos\alpha = n\cos\beta$$

then

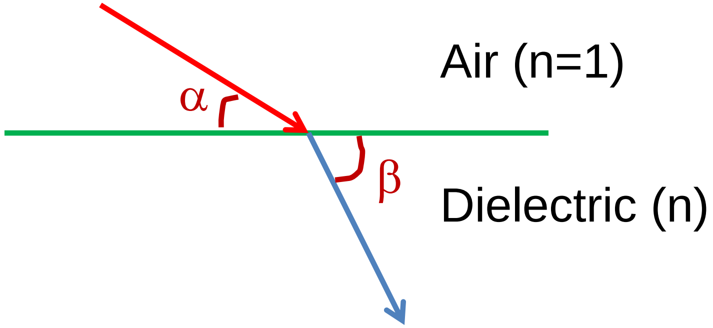
$$\sin\theta_1 = \sin\theta_2$$

$$(\theta_1 = \frac{\pi}{2} - \alpha, \theta_2 = \frac{\pi}{2} - \beta)$$

So if $\theta_1 = \pi/2$, $\theta_2 = \arcsin(1/n)$ the critical angle.

Solution: design an angle transformer with $\theta_1 = \theta_c, \theta_2 = \pi/2$

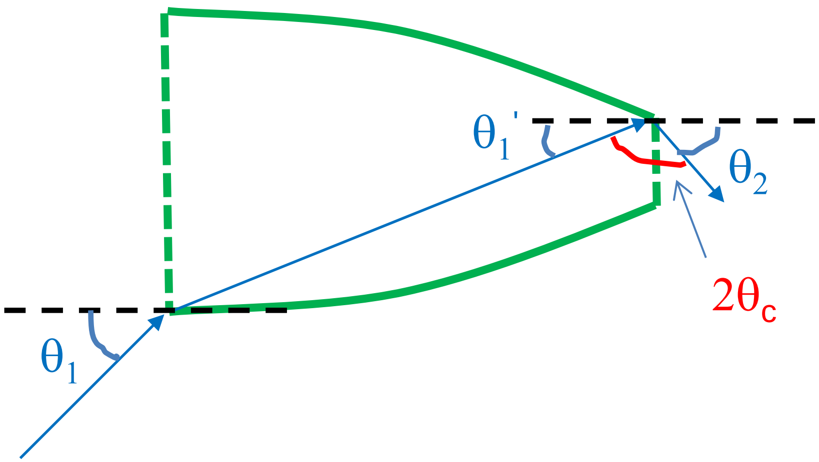
Then $C = \frac{1}{\sin\theta_c} = n$ (or n^2 for three dimensions)



How to design TIR concentrators

$\sin\theta_1 = n\sin\theta_1{}'$(inside the medium)

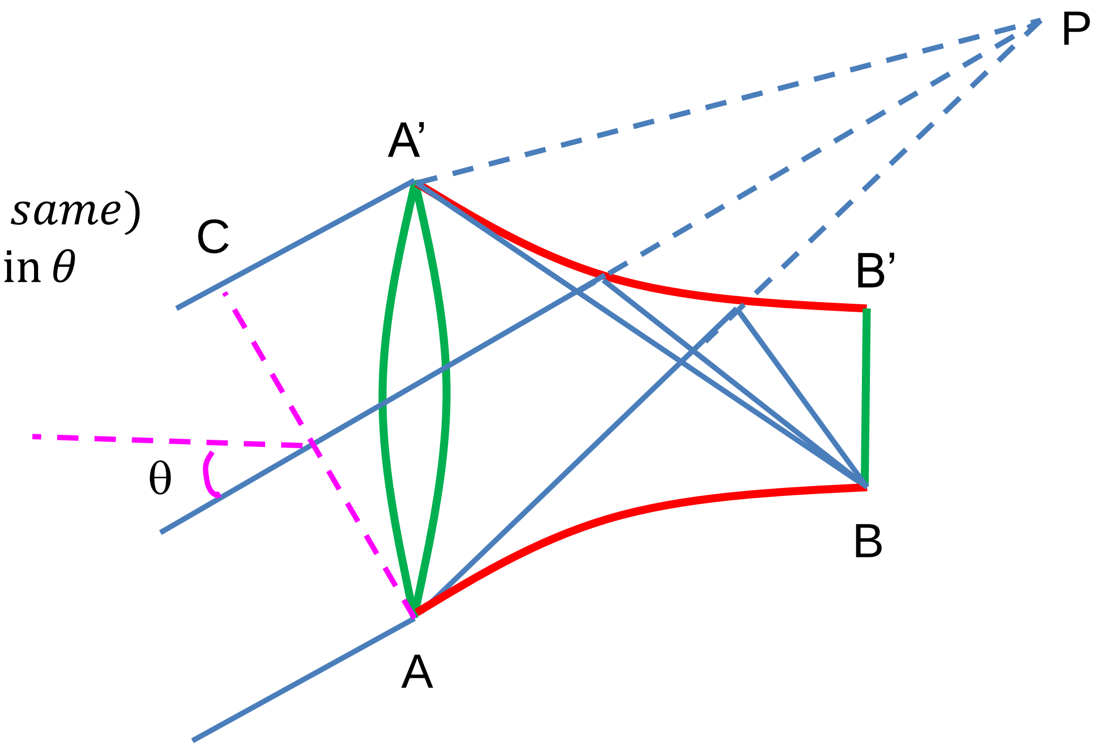
TIR condition $\theta_1' + \theta_2 + 2\theta_c = \pi, \theta_2 = \pi - 2\theta_c - \theta_1{}'$(or less)



Lens-mirror

String $\int ndl = constant$

$$AB' + B'B = CA' + A'B$$
$$(AB' and\ A'B\ \ are\ the\ same)$$
$$B'B = CA' = AA' \sin\theta$$



Some examples: n=1.5,$\theta_c = 42^{\circ}$

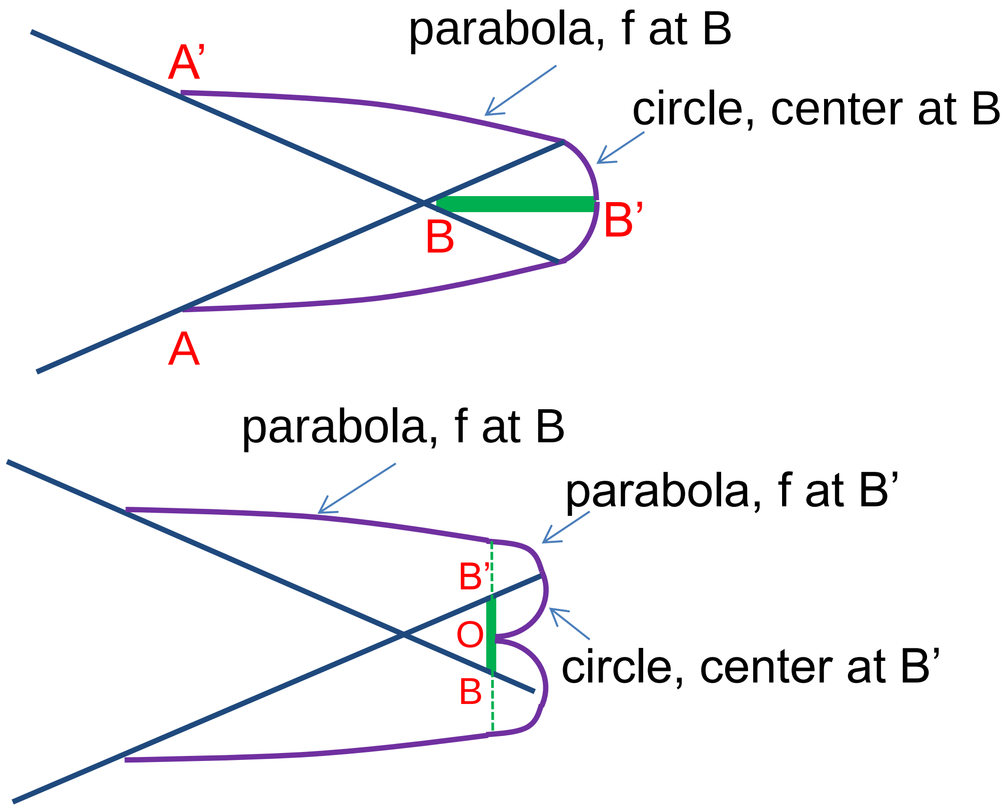
$\theta_2 = \pi/2$(90°),$\theta_1' \leq 6^{\circ}, \theta_1 < 9^{\circ}$

$\theta_1 = 30^{\circ}, \theta_1' = 19.5^{\circ}$,$\theta_2 = 96^{\circ} - 19.5^{\circ} = 76.5^{\circ}$

$$C = \frac{\sin\theta_2}{\sin\theta_1} = 0.97\left(\frac{1}{\sin\theta_1'}\right) = 0.97\,\frac{n}{\sin\theta_1} = 0.97\,C_{max} = 2.9$$

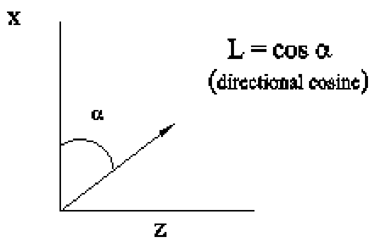
So an $f/1$ lens + TIR secondary = $f\# = \frac{0.5}{1.5*.97} = 0.34$ That's really fast.

Examples

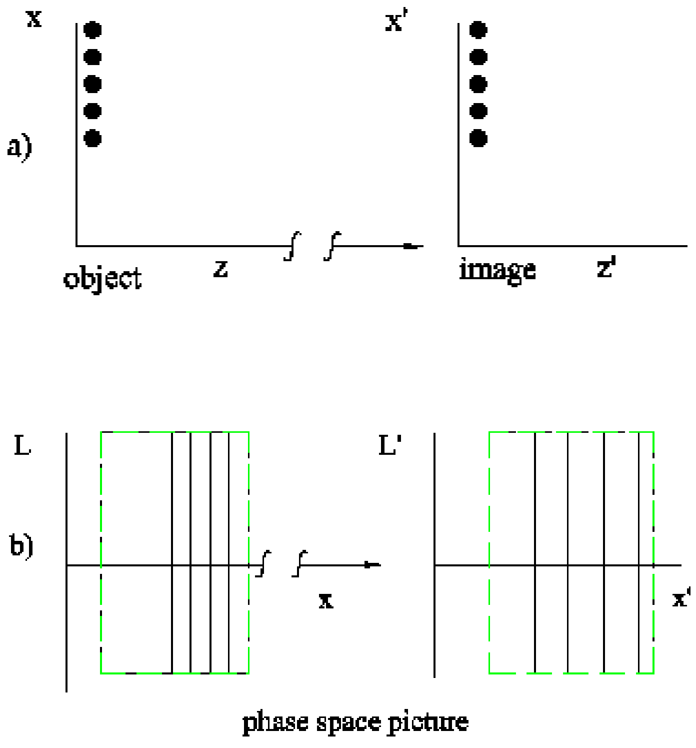

parabola, f at B
circle, center at B
A'
B'
B
A
parabola, f at B
parabola, f at B'
B'
O
B
circle, center at B'

Analogy of Fluid Dynamics and Optics

fluid dynamics	optics
phase space (twice the dimensions of ordinary space)	general etendue
positions	positions
momenta	directions of light rays multiplied by the index of refraction of the medium
incompressible fluid	volume in “phase space” is conserved

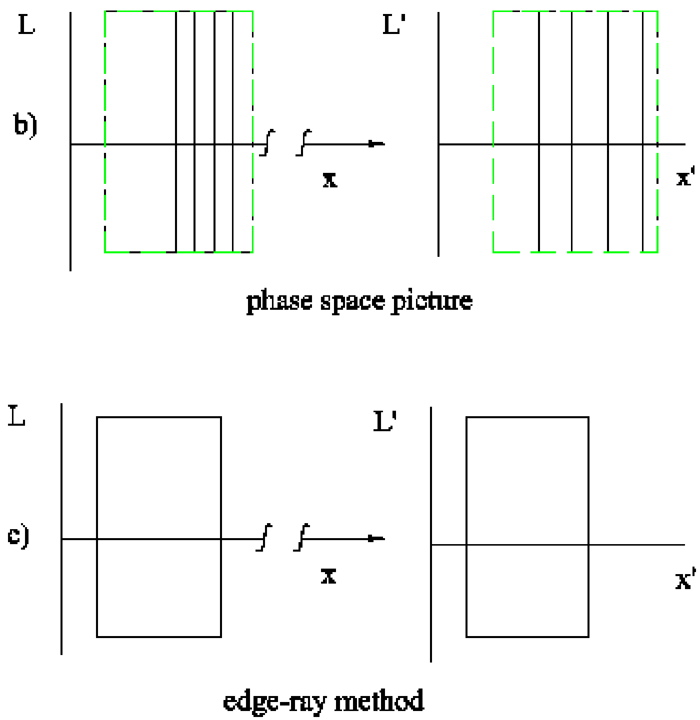


Imaging in Phase Space



- Example: points on a line.
  - An imaging system is required to map those points on another line, called the image, without scrambling the points.
- In phase space
  - Each point becomes a vertical line and the system is required to faithfully map line onto line.

Edge-ray Principle



- Consider only the boundary or edge of all the rays.
- All we require is that the boundary is transported from the source to the target.
  - The interior rays will come along . They cannot “leak out” because were they to cross the boundary they would first become the boundary, and it is the boundary that is being transported.

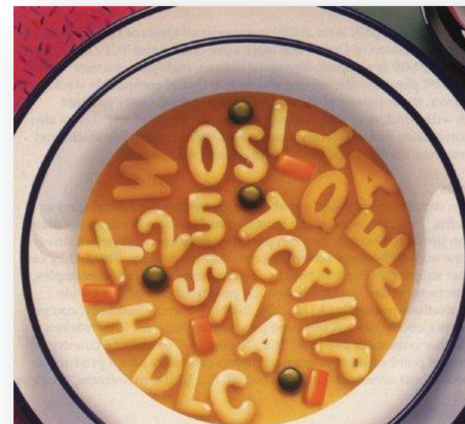
Edge-ray Principle

- It is very much like transporting a container of an incompressible fluid, say water.
- The volume of container of rays is unchanged in the process.
  - conservation of phase space volume.
- The fact that elements inside the container mix or the container itself is deformed is of no consequence.

Edge-ray Principle

- To carry the analogy a bit further, suppose one were faced with the task of transporting a vessel (the volume in phase-space) filled with alphabet blocks spelling out a message. Then one would have to take care not to shake the container and thereby scramble the blocks.
- But if one merely needs to transport the blocks without regard to the message, the task is much easier.

Non-Imaging Optics



Edge-ray Principle

- This is the key idea of nonimaging optics
- This leads to one of the most useful algorithms of nonimaging Optics.
- We shall see that transporting the edges only, without regards to interior order allows attainment of the *sine law of concentration limit.*

Flowline Method

Flowline Method

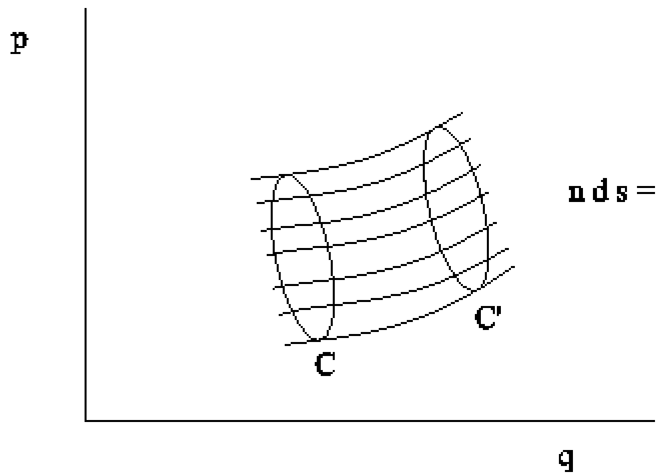
Then $\int \vec{J} \cdot d\vec{A}$ is conserved $=> Div\,\vec{J} = 0$
Design principle: placing the reflector along the lines of $\vec{J}$ does not disturb the flow.

This proposal, originally a conjecture is true for 2-D designs and for some 3-D designs of sufficient symmetry, as usual we teach by example: we start with Lambertian (black body) source, find the flow lines, examine the resulting designs and their applications.

Sphere: flow line are radial, design works in 2D and 3D.

Line(disc):How to find the flow lines

Phase Space Invariants



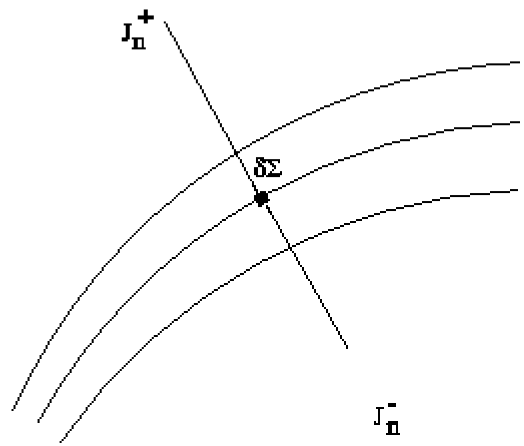
$n\,ds = \Sigma_i\, p_i\, dq_i$

1. $\oint p_i \;\; dx_i$

2. $\sum_{i\neq j} \int^{(4)} dp_i\, dp_j\, dx_i\, dx_j$ [important for nonimaging optics]

3. $\int^{(6)} dp_1\, dp_2\, dp_3\, dx_1\, dx_2\, dx_3$ [important for Liouville's Theorem]

Flowline Method



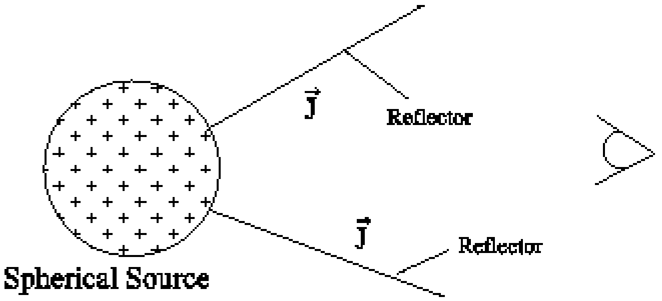
Flowline Method

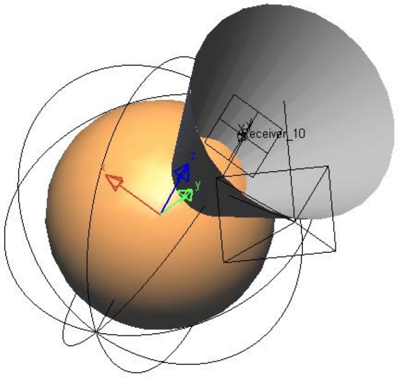
Let $\hat{n} = (L, M, N)$ unit vector, then, recall the Jacobean $dL\, dM = N d\Omega$,$d\Omega =$ dA $on\ a\ unit\ sphere$, $d\vec{A}_z = NdA = dLdM$

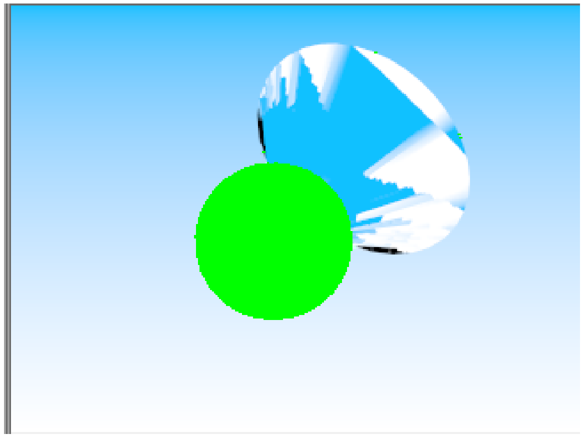
So in general $\vec{J} = \int n^2 \hat{n} d\Omega$, for the time being, keep $n = 1$.

So we can think of the flow line as the average direction of $\vec{J}$, works for a sphere.

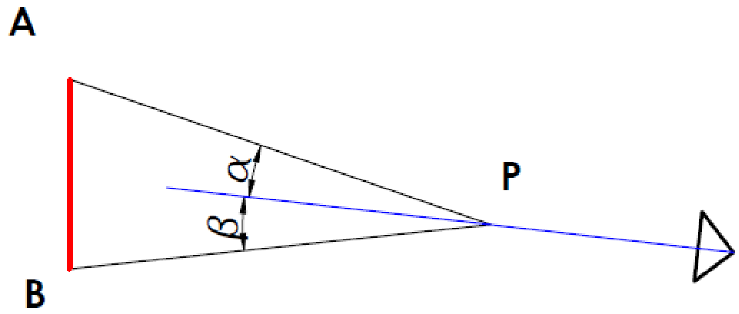
Try for a disc (line).



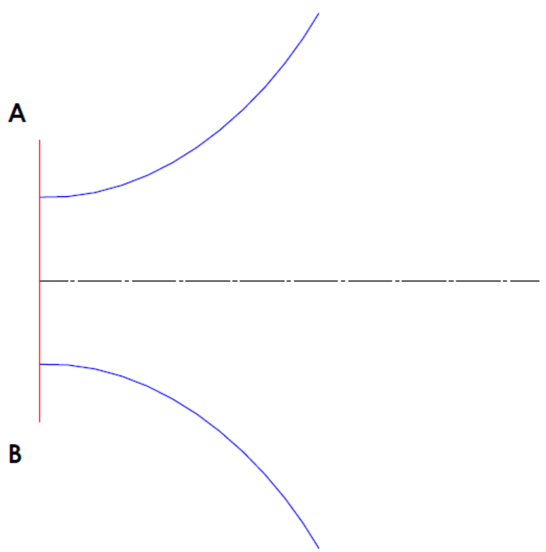


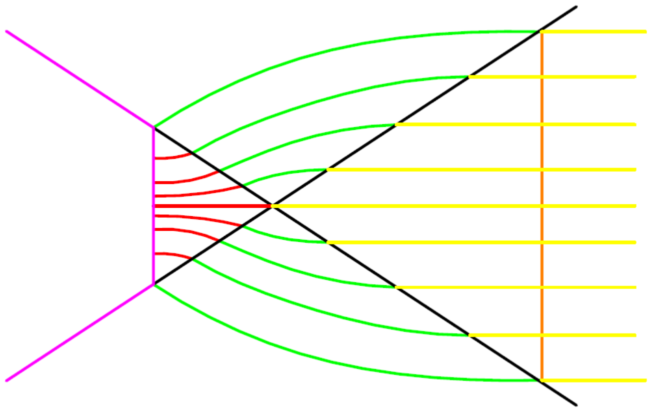


Flow line designs

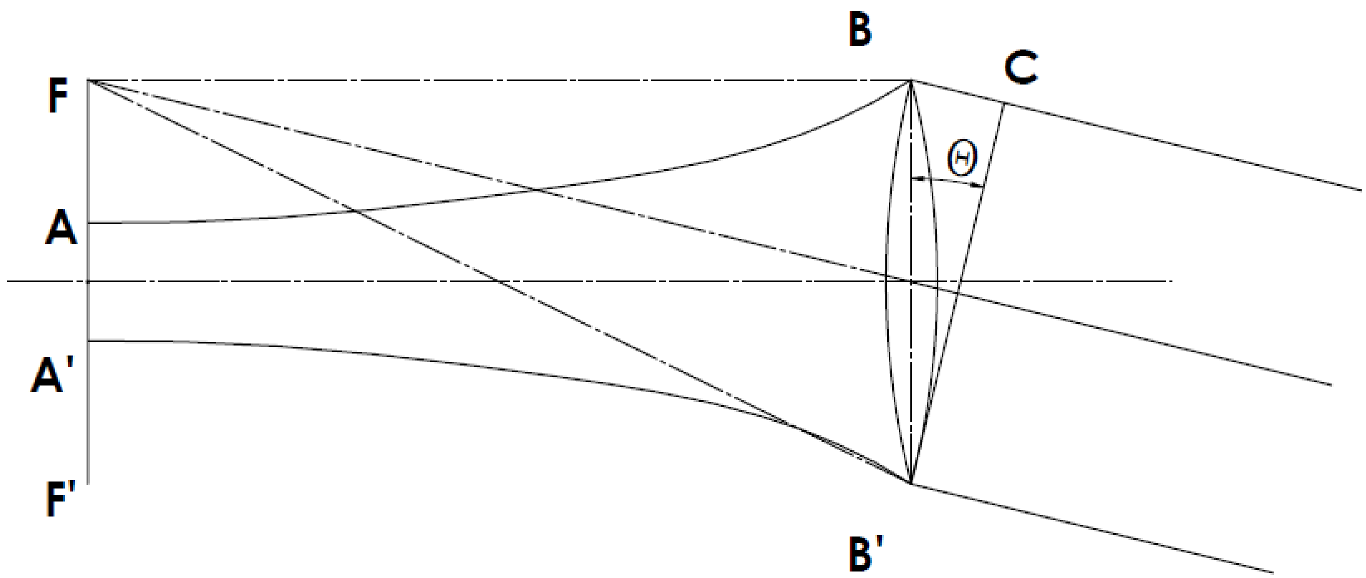


It is a property of the hyperbola that the angle to the forci (A,B) are equal, Consider





How to turn a slow lens to a fast lens - flow line



Fermat $BC + BF = B'F = BF'$
Hyperbola $B'F - BF = AF - A'F = AA'$
$BC = AA'$ but $BC = BB' sin\theta$

$$AA' = BB' sin\theta$$

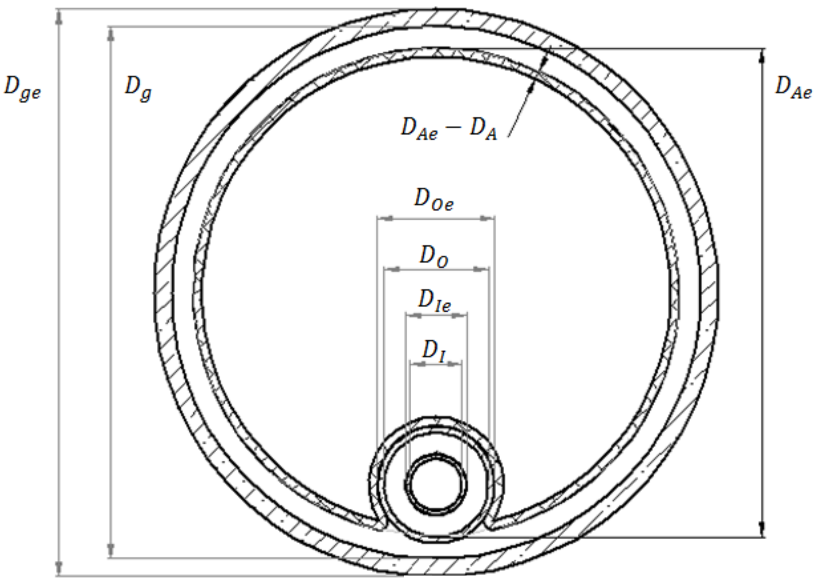
UCMERCED

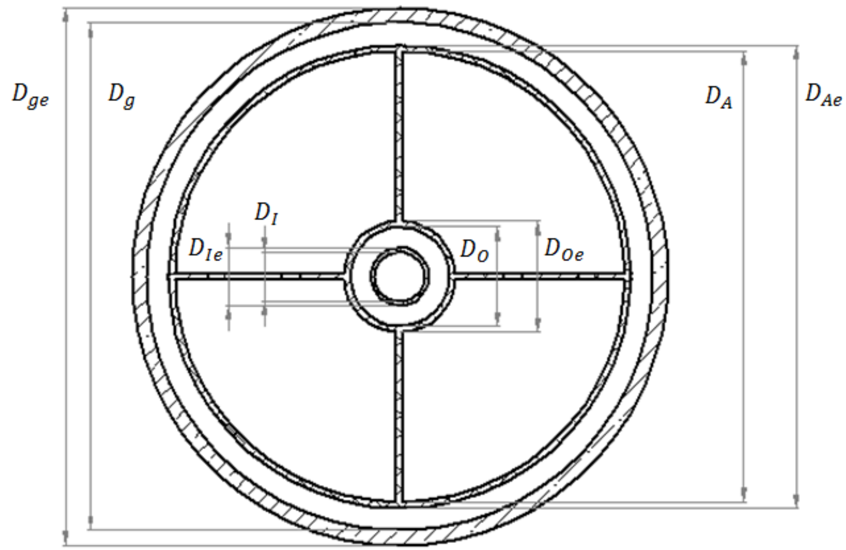
Highlight Project—Solar Thermal

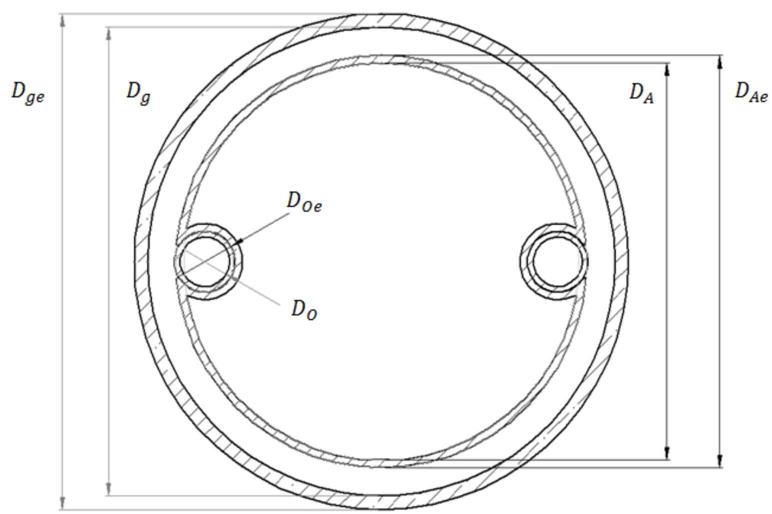
- UC Merced has developed the External Compound Parabolic Concentrator (XCPC)

- XCPC features include:
  - Non-tracking design
  - 50% thermal efficiency at 200°C
  - Installation flexibility
  - Performs well in hazy conditions
- Displaces natural gas consumption and reduces emissions
- Targets commercial applications such as double-effect absorption cooling, boiler preheating, dehydration, sterilization, desalination and steam extraction

UC Merced 250°C Thermal Test Loop

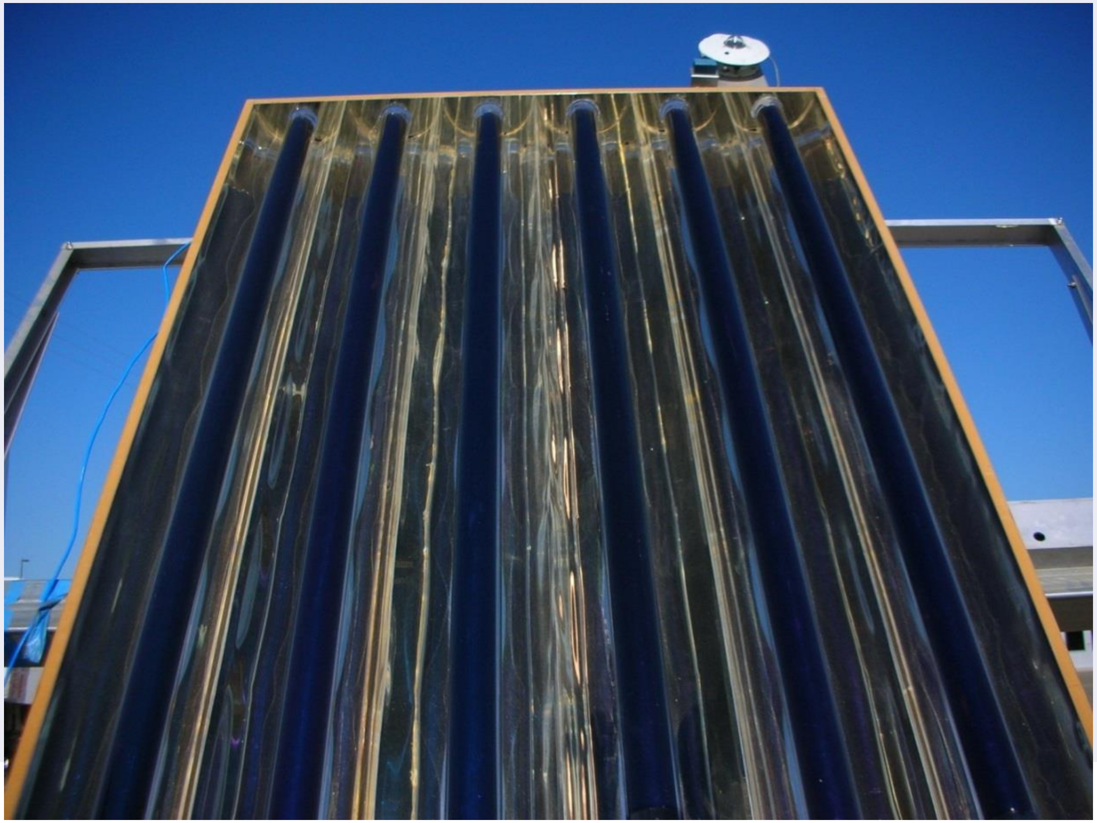

D_{ge}
D_g
D_{Ae}
$D_{Ae} - D_A$
D_{Oe}
D_O
D_{Ie}
D_I


D_{ge}
D_g
D_A
D_{Ae}
D_I
D_{Ie}
D_O
D_{Oe}

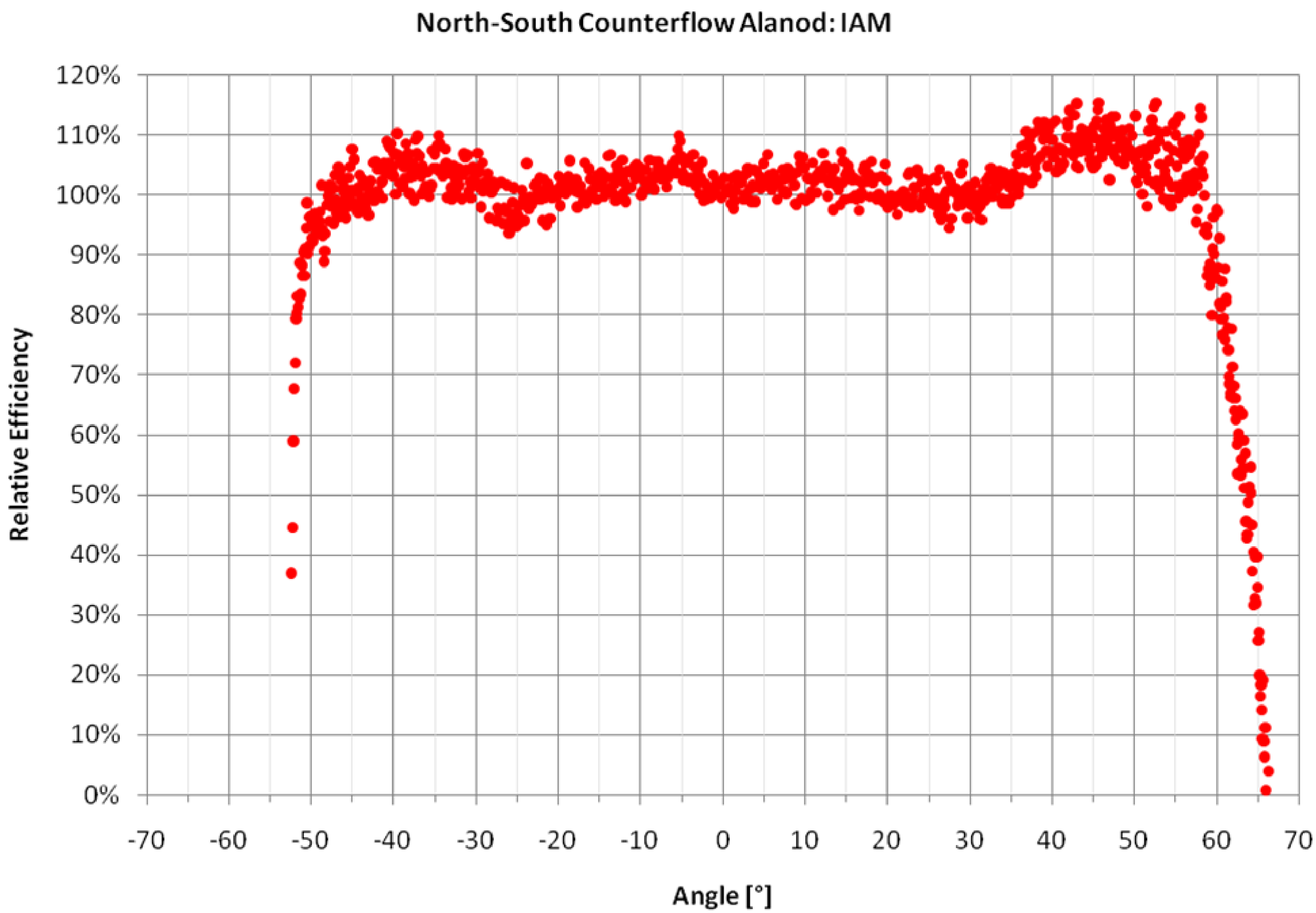

D_{ge}
D_g
D_A
D_{Ae}
D_{Oe}
D_O

Testing

- Efficiency (80 to 200 °C)
- Optical Efficiency (Ambient temperature)
- Acceptance Angle
- Time Constant
- Stagnation Test

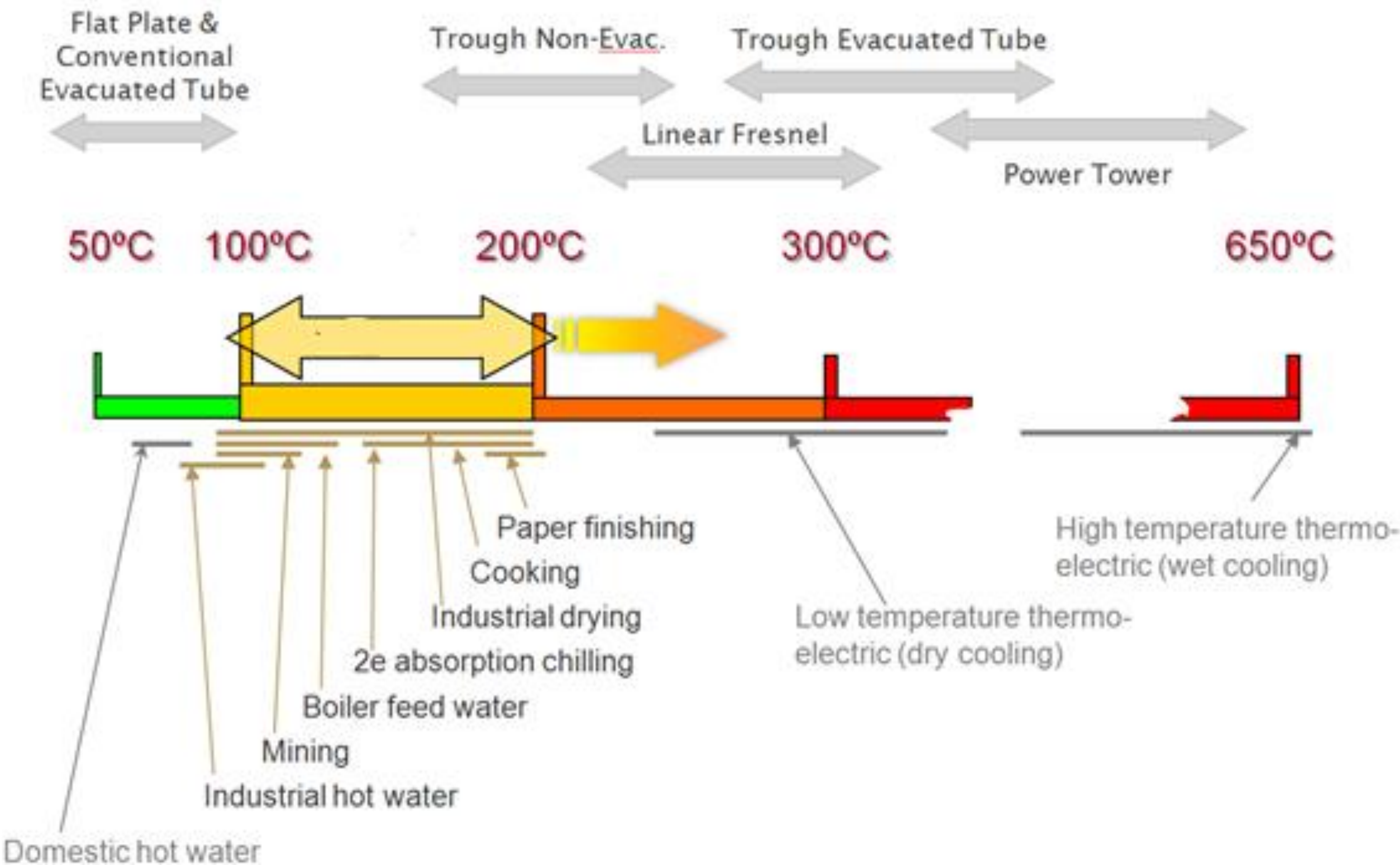


Acceptance Angle



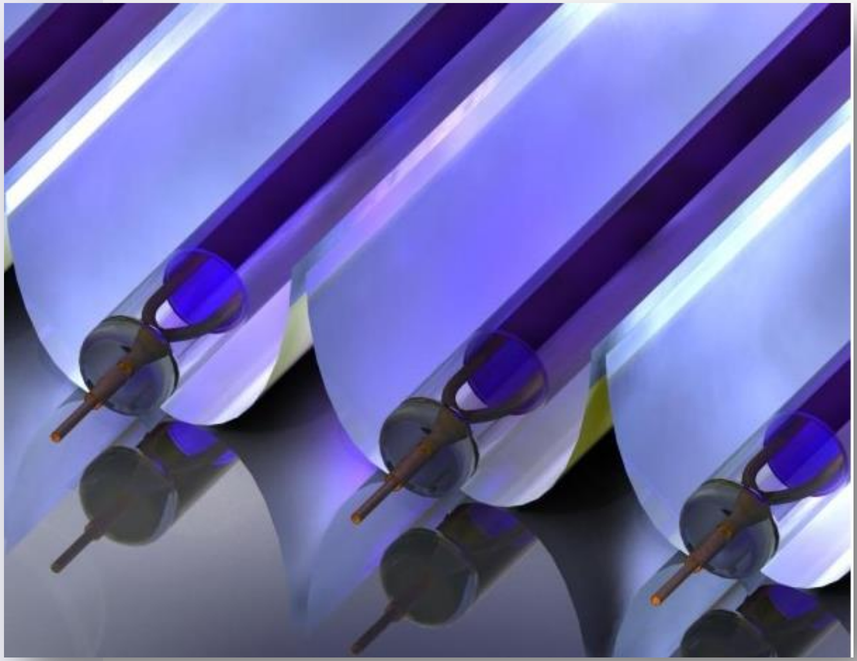
Thermodynamically Efficient Solar Cooling

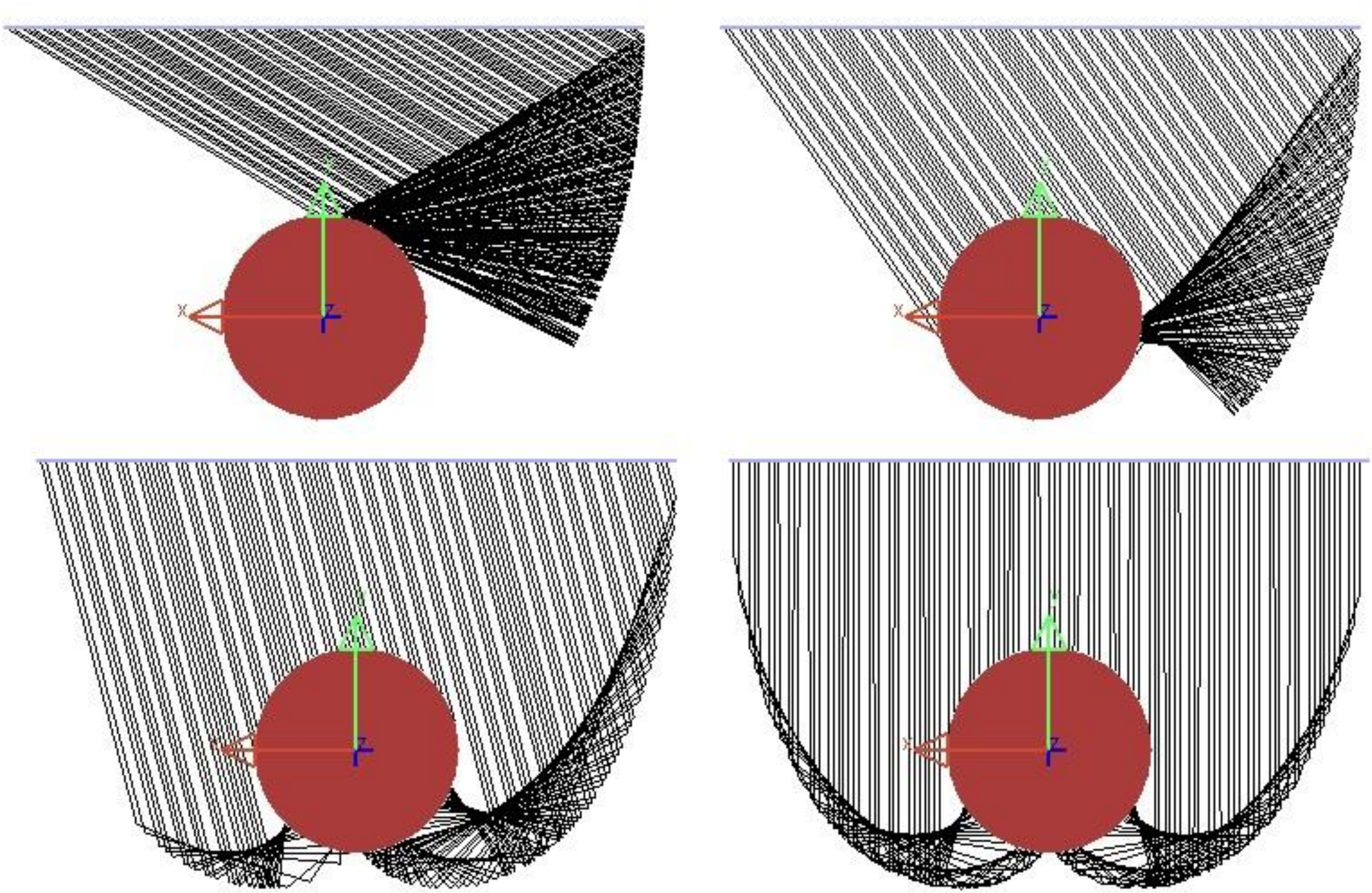
- Solar Cooling
  - Using energy from the sun to provide space cooling / refrigeration
  - Well matched supply/load (i.e. High cooling demand on sunny days)
  - If roof deployed, energy that would heat up building is diverted for cooling
- Barriers
  - Efficient cooling machines (double effect absorption chillers) require high temperatures around 180 C
  - Tracking collectors are problematic
  - Absorption chillers do not respond well to natural variability of solar insolation
- Solution
  - Gas/Solar hybrid absorption chillers
  - Development of new high temperature, fixed solar collector at UC Merced


Flat Plate &
Conventional
Evacuated Tube
Trough Non-Evac.
Trough Evacuated Tube
Linear Fresnel
Power Tower
50ºC
100ºC
200ºC
300ºC
650ºC
Paper finishing
Cooking
Industrial drying
2e absorption chilling
Boiler feed water
Mining
Industrial hot water
Domestic hot water
Low temperature thermo-
electric (dry cooling)
High temperature thermo-
electric (wet cooling)

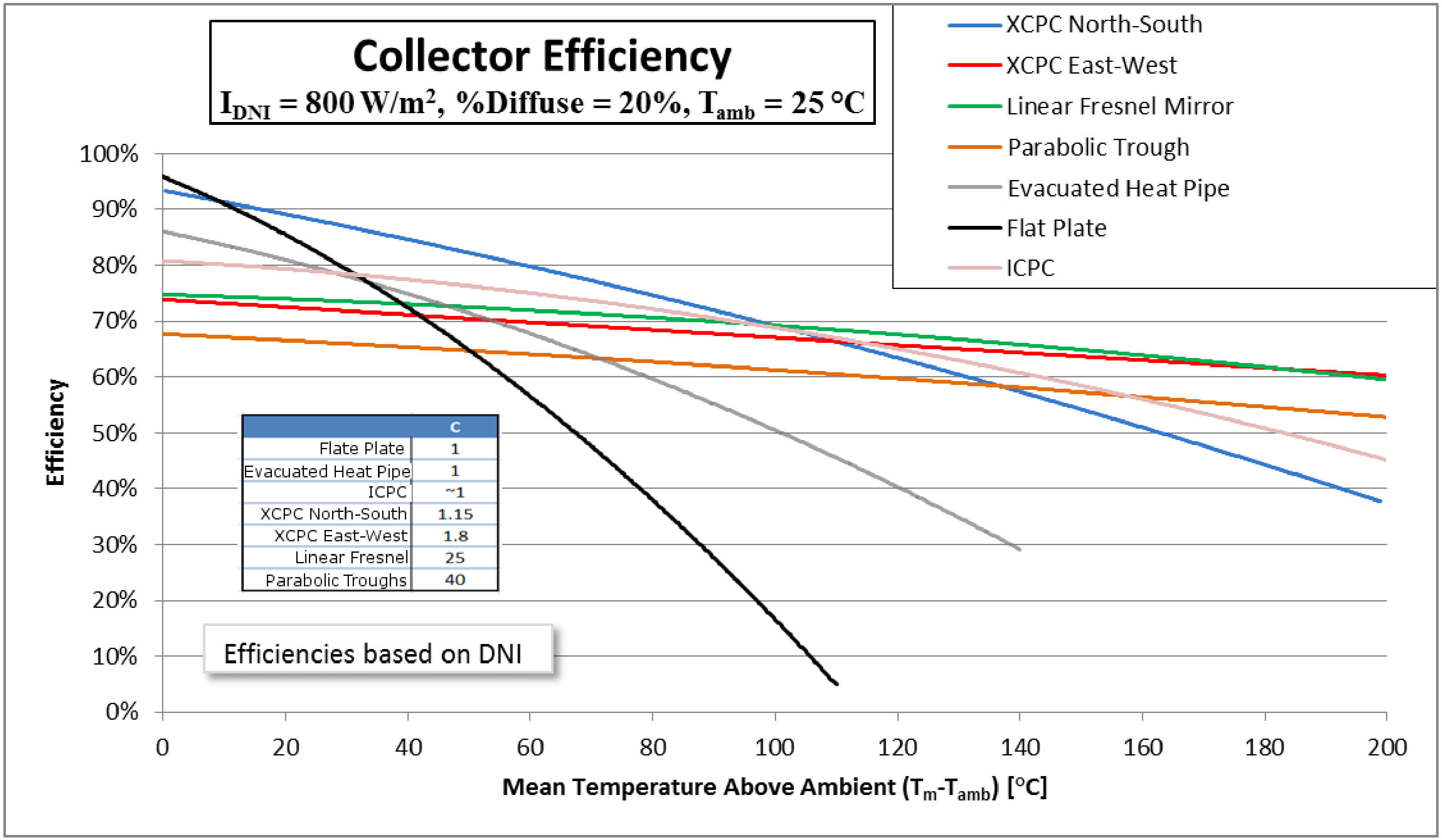
XCPC

- Non-imaging optics:
  - External Compound Parabolic Concentrator (XCPC)
  - Non-tracking
  - Thermodynamically efficient
  - Collects diffuse sunlight
  - East-West and North-South designs





Performance Comparison

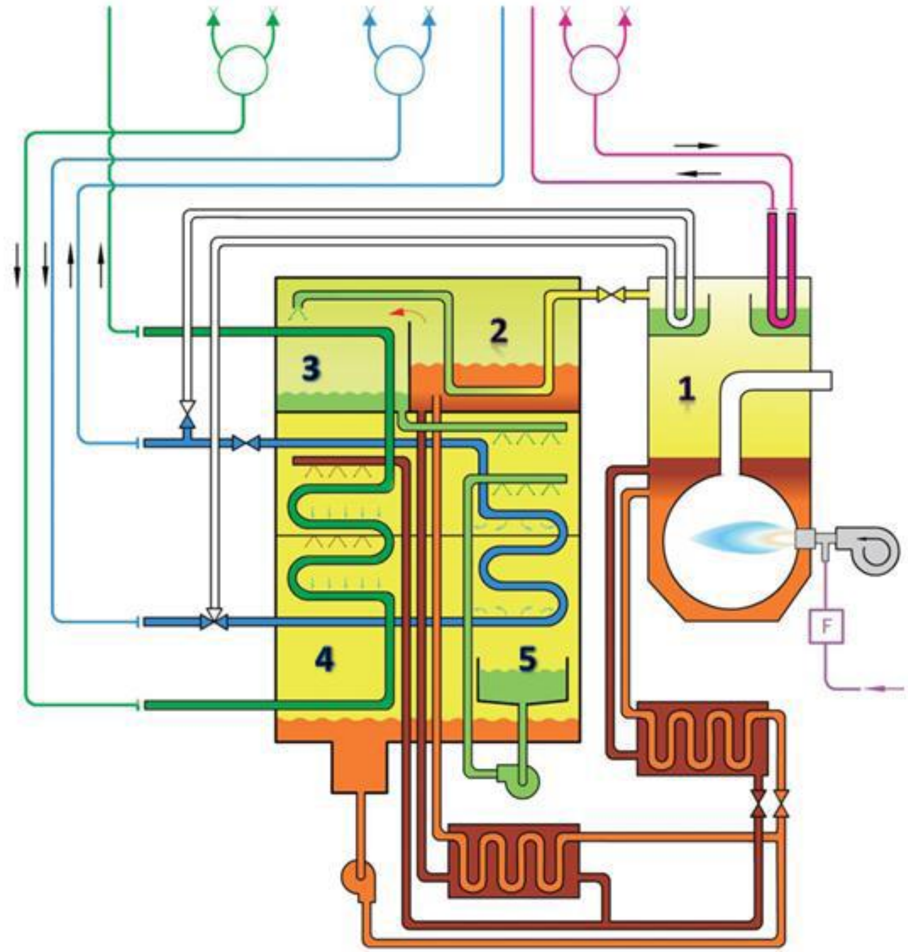


	C
Flate Plate	1
Evacuated Heat Pipe	1
ICPC	~1
XCPC North-South	1.15
XCPC East-West	1.8
Linear Fresnel	25
Parabolic Troughs	40

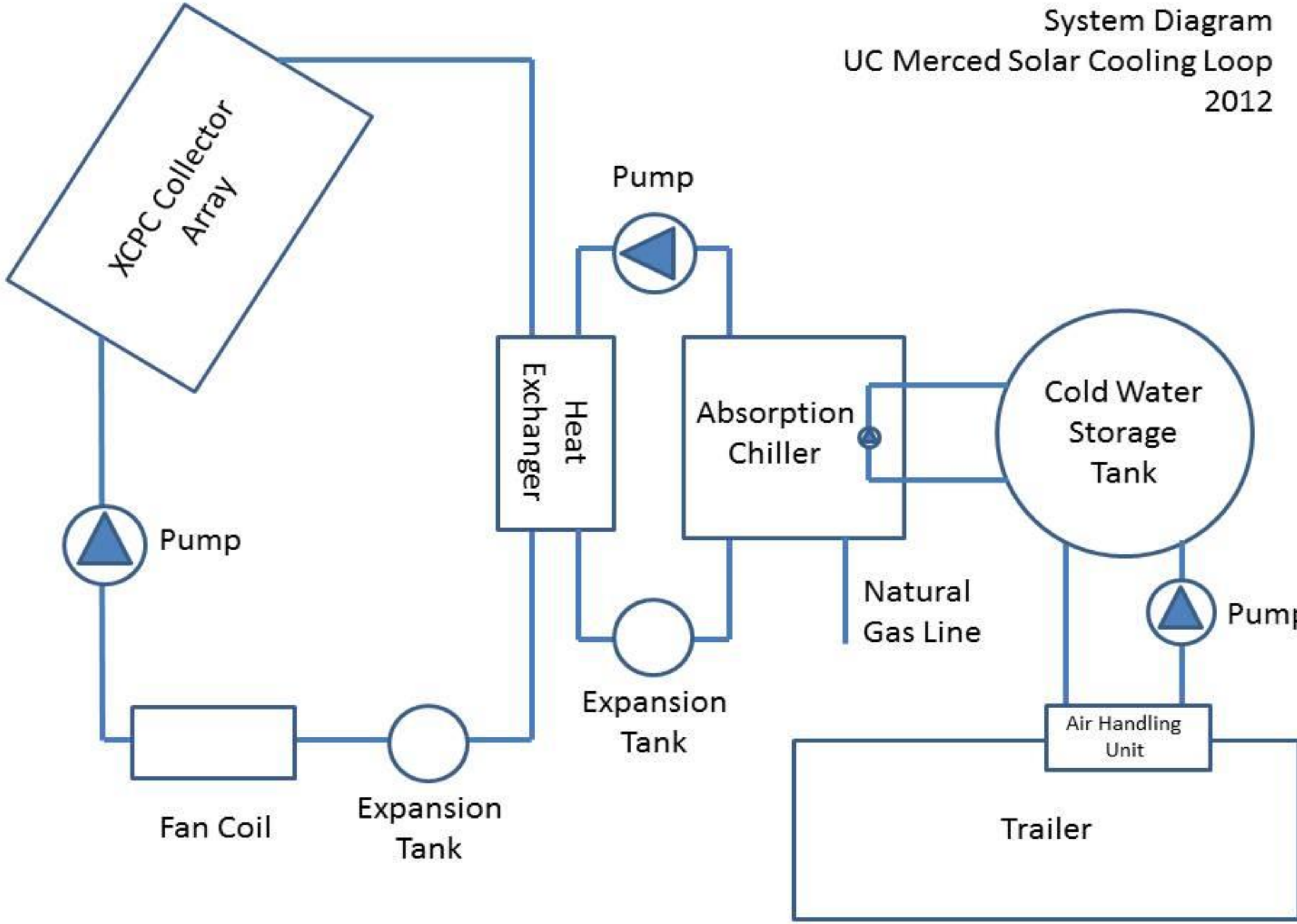
Solar Cooling at UC Merced

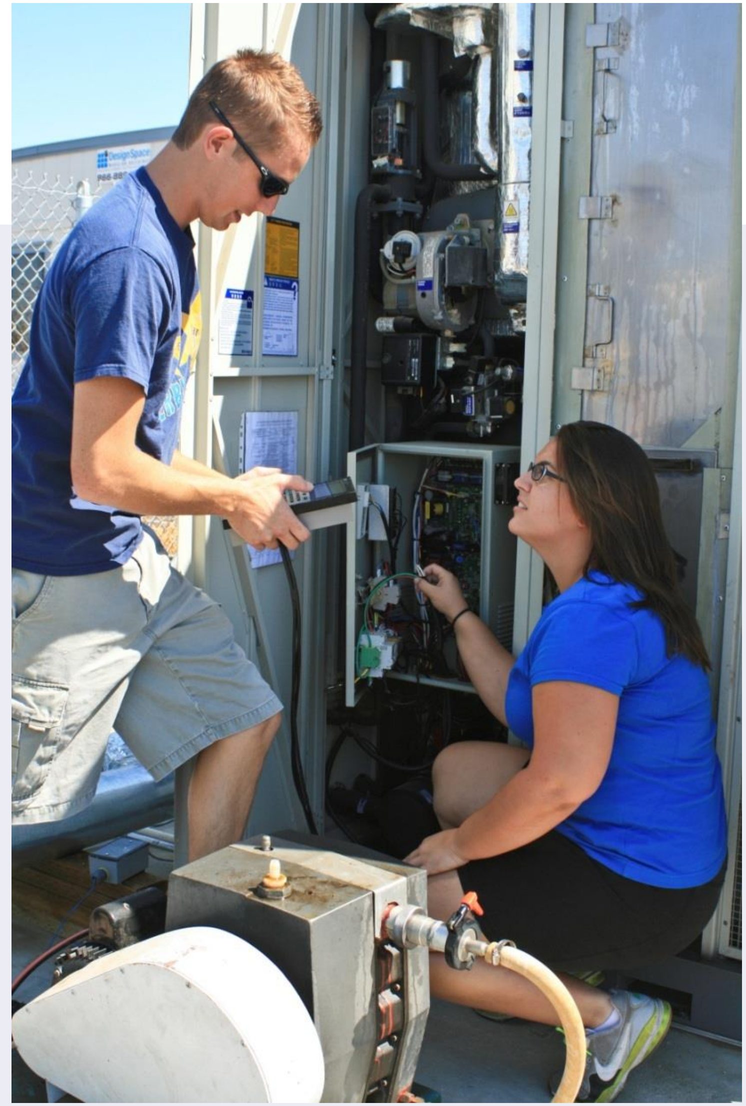
- Collectors
  - 160 North/South XCPCs
  - Concentration ratio ~ 1.18
  - 50 m2 inlet aperture area
- Chiller (BROAD Manufacturing)
  - 6 ton (23 kW) Lithium Bromide Absorption Chiller
  - Double effect (COP ~ 1.1)
  - Hybrid solar / natural gas powered

XCPC Array at UC Merced

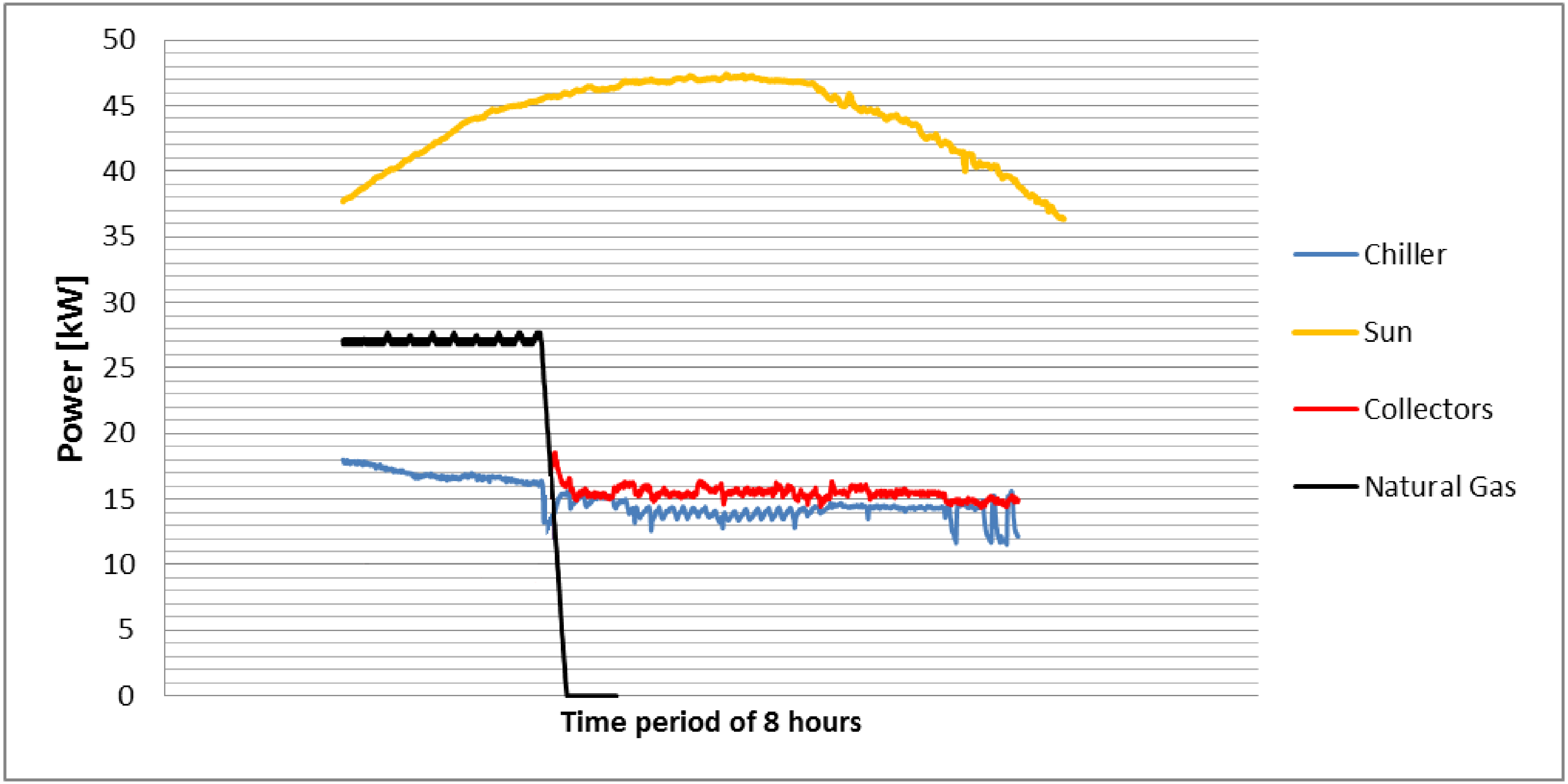

1
2
3
4
5
F

System Diagram
UC Merced Solar Cooling Loop
2012

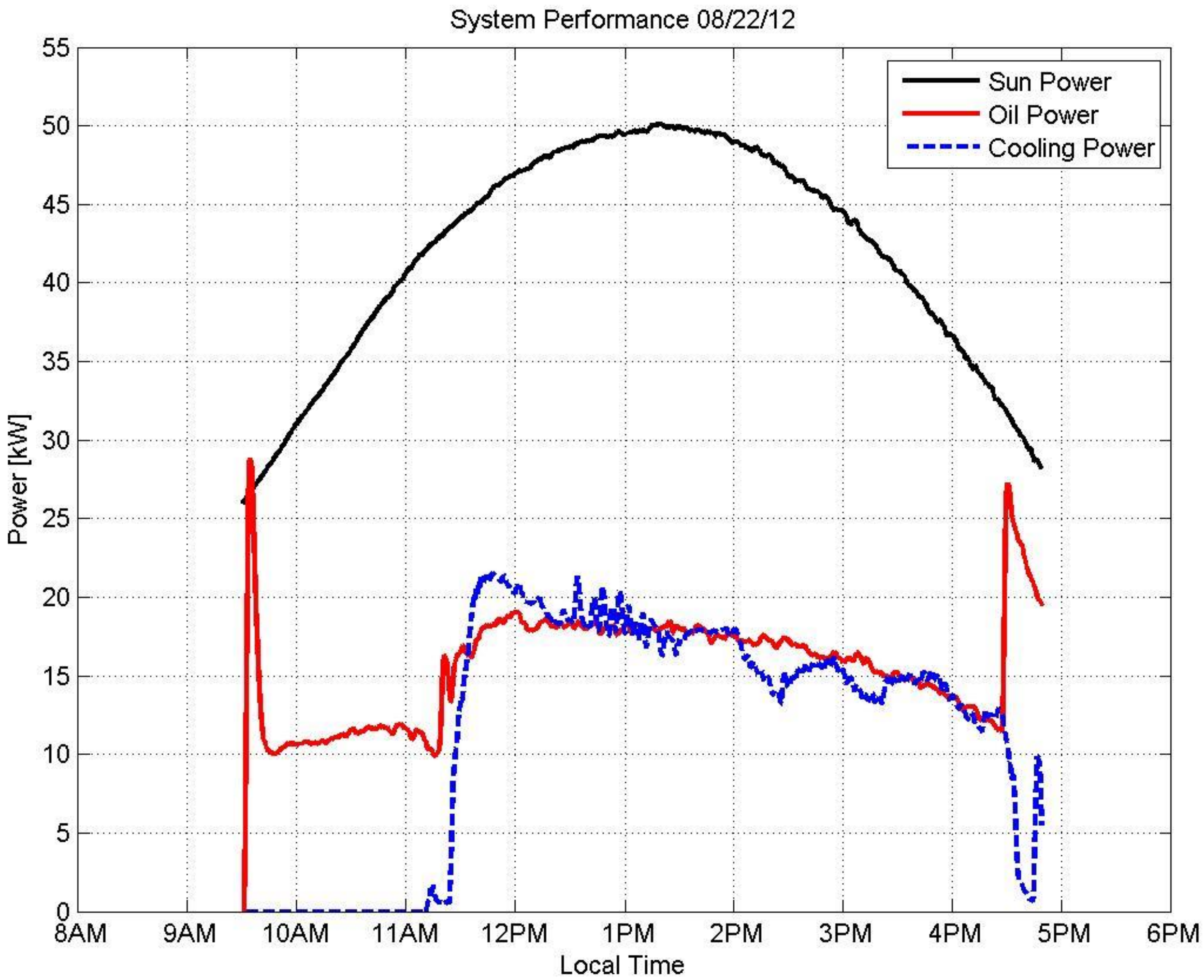




Power Output of Solar Cooling 2011

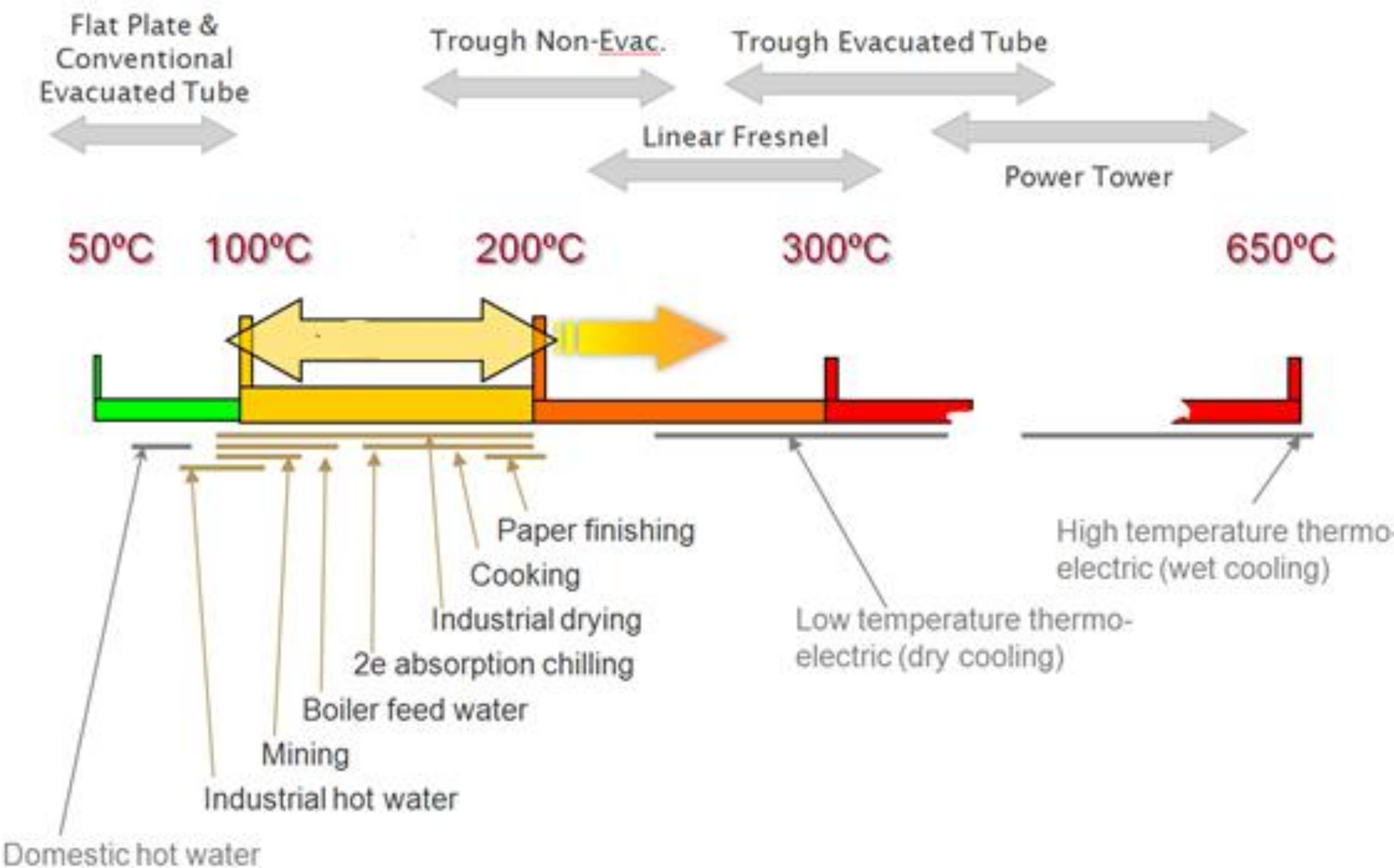


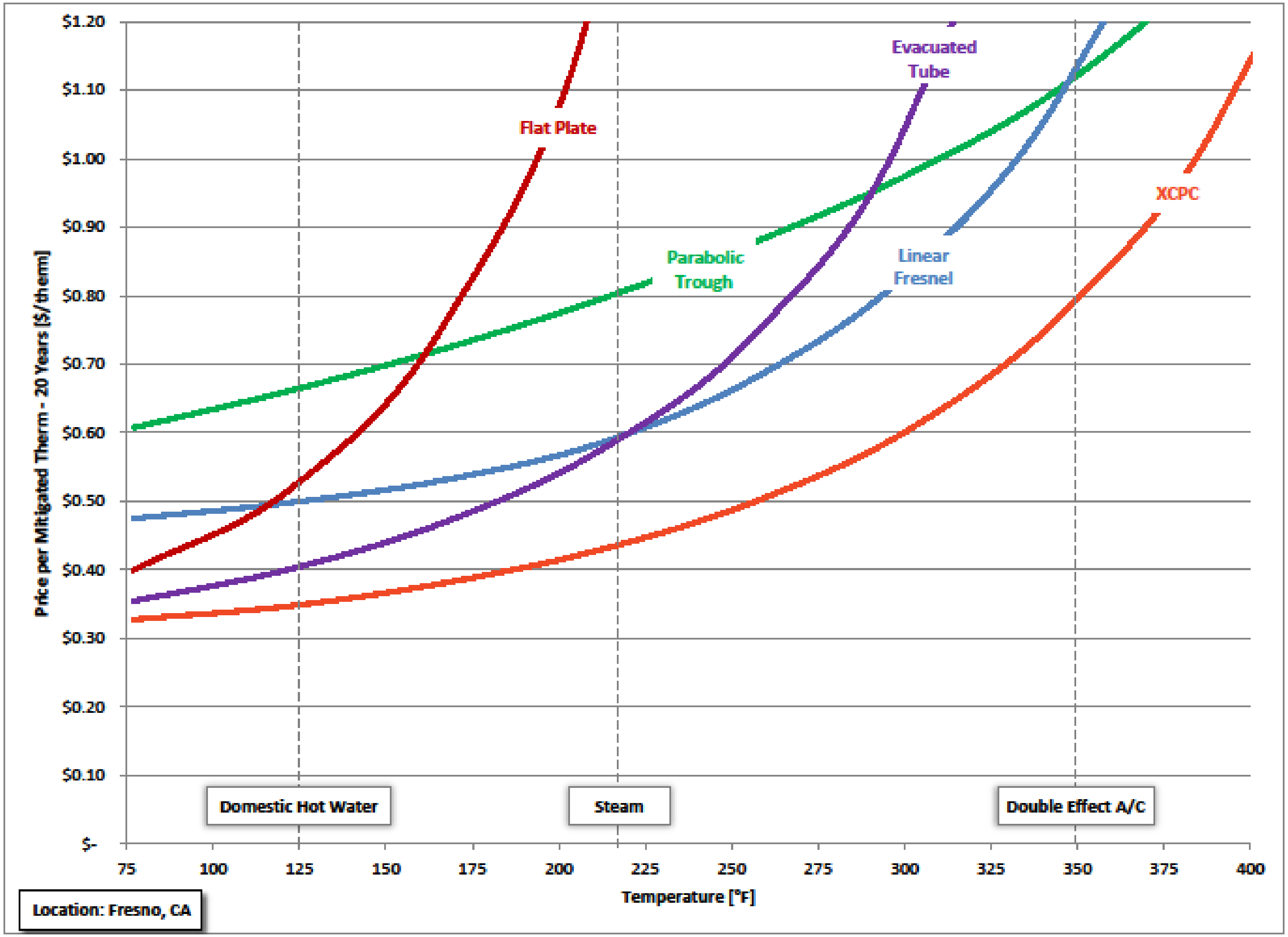
Power Output of Solar Cooling 2012



In Summary

- XCPC
  - Fixed, high temperature solar thermal collector
  - High thermodynamic efficiency
  - Collects diffuse light
  - Flexible installation
- UC Merced Solar Cooling Project
  - 160 North/South XCPCs (~50 m2)
  - 6 ton (23 kW) Li-Br Double Effect Absorption chiller
  - Natural gas-powered chiller during system warm-up
  - Direct solar powered cooling for about 6 hours
  - Extended solar cooling for about 2 hours
  - Average Daily Solar COP of about 0.38


Flat Plate &
Conventional
Evacuated Tube
Trough Non-Evac.
Trough Evacuated Tube
Linear Fresnel
Power Tower
50ºC
100ºC
200ºC
300ºC
650ºC
Paper finishing
Cooking
Industrial drying
2e absorption chilling
Boiler feed water
Mining
Industrial hot water
Domestic hot water
Low temperature thermo-
electric (dry cooling)
High temperature thermo-
electric (wet cooling)



$1.20
$1.10
$1.00
$0.90
$0.80
$0.70
$0.60
$0.50
$0.40
$0.30
$0.20
$0.10
$-
Price per Mitigated Therm - 20 Years [$/therm]
Flat Plate
Evacuated Tube
Parabolic Trough
Linear Fresnel
XCPC
Domestic Hot Water
Steam
Double Effect A/C
75
100
125
150
175
200
225
250
275
300
325
350
375
400
Temperature [°F]
Location: Fresno, CA

XCPC Applications

- **Absorption Chillers**
- Adsorption Chillers
- Desiccant Cooling
- Heat Driven Electrical Power Generation
- Steam Cycle Based Products
- Stirling Cycle Based Products
- Heat Driven Water Treatment Technology
- Membrane Distillation
- Heat Driven Industrial Process
- Technology feasibility
- Economic Competitiveness
- Market Potential
- Time to commercialization

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Demonstrated Performance

10kW Array Gas Technology Institute

Conceptual Testing
SolFocus & UC Merced

10kW test loop NASA/AMES