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Distributed Storage Investment in Power Networks

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Abstract— **The value created by aggregating behind-the-meter distributed energy storage devices for grid services depends on how much storage is in the system and the power network operation conditions. To understand whether market-driven distributed storage investment will result in a socially desirable outcome, we formulate and analyze a network storage investment game. By explicitly characterizing the set of Nash equilibria (NE) for two examples, we establish that the uniqueness and efficiency of NE depend critically on the power network conditions. Furthermore, we show it is guaranteed that NE support social welfare for general power networks, provided we include two modifications in our model. These modifications suggest potential directions for regulatory interventions.**

I. Introduction

From the conventional power system wisdom that storing electricity on a large scale is prohibitively expensive to the widely accepted understanding that electric storage will play key roles in grids with deep renewable penetration, the electricity industry's view on electric energy storage, especially those based on batteries, has dramatically changed over the last two decades. This is driven by profound technological and economic trends including the stunning reduction of the costs of battery systems and the advance of power electronics for programmable inverters. In this process, new storage capacities are being connected to the grid, at a rate that grows rapidly from year to year [1].Among the newly installed storage capacities, an increasing portion consists of behind-the-meter distributed storage devices. For instance, behind-the-meter storage deployment in the U.S. sees a 79% year-over-year growth in 2017 [2].

At today's capital and installation costs, behind-the-meter batteries often can only be cost-effective if they provide multiple sources of revenue [3]. An important class of revenue streams involve operating behind-the-meter batteries to provide grid services, such as load shifting and peak shaving. Thus investment decisions for distributed energy storage are in part driven by the expectation of the revenues generated in these use cases. Meanwhile, value created in these use cases are usually tied to the operation conditions of the power network, which depend in turn on how much storage is connected to the grid. Therefore a fundamental question regarding distributed energy storage investment is: *will this market-driven closed-loop dynamics lead to a socially desirable level of storage investment?*

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A. Contributions and paper organization

The paper examines this question in a specific context where the distributed storage capacities at each distribution feeder[1] are aggregated to participate in the wholesale (transmission) electricity market. In the wholesale market, a transmission-constrained multi-period economic dispatch problem is solved to determine the operation of the grid and the storage devices, and the locational marginal prices. These prices then determine the payments to the storage owners.

To model the storage investment decisions of a potentially large number of users residing on each distribution feeder, we formulate a storage investment game with a continuum of players (Section II) where each player determines whether to install storage by considering the capital and installation costs, the payment from participating in the wholesale market, and an outside value for installing storage. In two examples with different power network settings, we establish qualitatively different results on the uniqueness and efficiency of the Nash Equilibrium (NE) of the investment game by providing an explicit characterization of the set of NE and comparing to several benchmark solutions (Section IV and Section V). In particular, our results for a complementary storage investment setup (Section V), that NE are not unique and may not support social welfare, are in stark contrast to prior results on storage investment games [4], [5] where the power network constraints and the impact of the storage capacities on the prices are not considered. Despite these negative results, we then show in Section VI that even in general network, NE of the storage investment game are guaranteed to support social welfare, provided that we impose two modifications to our original setup. We then discuss potential real world implementations of these modifications as changes to the aggregator business model and the wholesale market bidding formats. The paper is concluded in Section VII.

B. Literature

The power systems literature on electric energy storage can be broadly categorized into storage operation and planning. Studies on storage operation (see e.g. [6], [7] and references therein) aim to devise efficient control rules for energy storage devices, given some fixed storage capacities. The storage planning literature (see e.g. [8], [9]) usually assume a storage operation model and then address the questions of storage sizing (i.e., how much storage to build) and

[1]We focus on behind-the-meter distributed storage devices because correctly aligned incentives are crucial for this segment of the energy storage industry to properly grow. In contrast, front-of-meter storage projects are often centrally planned and procured by utility companies.

placement (i.e, how to allocate some total storage capacity over the power network).

The problem of energy storage investment can be thought of as a kind of planning study. However, different from centralized planning studies where an optimization is solved to determine the optimal storage capacities over the power network, we obtain the storage capacities as natural outcomes of the incentives in the storage investment game (through NE). Storage investment game has been investigated in prior studies [4], [5] where power network constraints and the dependence of the prices on the storage capacities are omitted. We demonstrate that qualitatively different results can be obtained when these factors are considered.

Analogous to the storage investment problem is the transmission investment problem (see e.g. [10]–[12]). Despite substantial differences between energy storage devices and transmission lines, energy storage can be modeled as a one-directional transmission line that enables sending energy to future time periods. As a consequence, intuitions developed for the transmission investment problem sometimes can be applied to the storage investment problem. For instance, our example on complementary storage investment is inspired by the impactful work on transmission storage investment of Joskow and Tirole [10].

II. Model

Consider a setting where small investors residing on the distribution feeders are represented by aggregators to participate in transmission power market (Fig. 1).

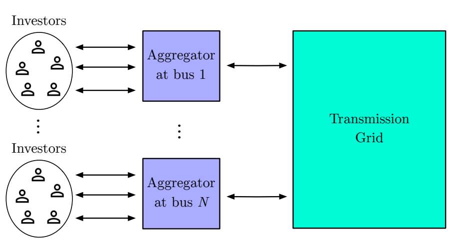


Fig. 1: Schematic for distributed storage investment

A. Transmission grid model

We consider a connected power transmission network with the set of buses denoted by $\mathcal{N}$, where the total number of buses is $N = |\mathcal{N}|$. For notational convenience, we assume each bus is connected to a distribution feeder and perhaps a power generation plant. In the context of storage investment and planning, we consider the operation of the transmission network over a finite horizon $\mathcal{T}$, where the total number of time periods is $T = |\mathcal{T}|$. Here the operation horizon may model hourly operation for a representative day or a longer period, capturing the typical seasonal variation of the grid operation data.

Denote the aggregate load, and controllable generation at bus $n \in \mathcal{N}$ and time $t \in \mathcal{T}$ by $d_{n,t}$ and $g_{n,t}$, respectively. The cost of generating power at bus n and time t is modeled as a convex quadratic function $c_{n,t}(g_{n,t})$, and the benefit for consuming power is modeled as a concave quadratic function $b_{n,t}(d_{n,t})$.

The storage capacity at bus $n \in \mathcal{N}$, made available by the aggregator at bus n to the transmission grid operator, is denoted by $S_n \geq 0$. The transmission grid operator determines the control of the aggregate storage at each bus $n \in \mathcal{N}$, denoted by $u_{n,t}$, where $u_{n,t} > 0$ represents charging and $u_{n,t} < 0$ represents discharging. The sequence of charging and discharging operations must satisfy the physical constraints of the energy storage. For simplicity, we utilize a stylized model for energy storage and focus on the energy constraints of the form

$$0 \leq \sum_{\tau=1}^{t} u_{n,\tau} \leq S_n, \quad t \in \mathcal{T}, \tag{1}$$

where we assume that the initial state of charge is zero.

The generation, load and storage operation determine the net power injection into the grid. For each time $t \in \mathcal{T}$, the vector of net power injection

$$p_t = g_t - d_t - u_t \in \mathbb{R}^N$$

must satisfies power flow constraints. Utilizing a standard linearized power flow model (i.e. DC approximation to AC power flow), we can write the power flow constraints as

$$p_t \in \mathcal{P} := \{x \in \mathbb{R}^N : \mathbf{1}^\top x = 0,\ Hx \leq \ell\}, \quad t \in \mathcal{T}, \tag{2}$$

where the first constraint enforces power balance over the grid and the second constraint ensures that the flow on each transmission line does not exceed its thermal capacity.

The transmission grid operator solves the following optimization, referred to as the *multi-period economic dispatch* problem, to determine the operation of the grid and the aggregate storage capacities:

$$\begin{aligned} J(S) := \min_{g,d,u,p} \quad & \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{N}} c_{n,t}(g_{n,t}) - b_{n,t}(d_{n,t}) && \text{(3a)} \\ \text{s.t.} \quad & p_t = g_t - d_t - u_t, \quad t \in \mathcal{T}, && \text{(3b)} \\ & p_t \in \mathcal{P}, \quad t \in \mathcal{T}, && \text{(3c)} \\ & 0 \leq L u_n \leq S_n \mathbf{1}, \quad n \in \mathcal{N}, && \text{(3d)} \end{aligned}$$

where matrix $L \in \mathbb{R}^{T\times T}$ encodes the battery dynamics and is defined as

$$L_{tt'} = \begin{cases} 1, & \text{if } t' \leq t, \\ 0, & \text{otherwise.} \end{cases}$$

The optimization identifies the optimal schedules for controllable generation, elastic load, and energy storage to minimize the overall operation cost for the grid, respecting the physical constraints imposed by the power network and the storage devices. As the solution to (3) depends on how much storage capacities are connected to the power grid, we denote the solution of (3) by $(g^\star(S), d^\star(S), u^\star(S), p^\star(S))$.

This optimization also defines a set of prices, referred to as *locational marginal prices* (LMP), that are used to clear the transmission power market. In particular, denote the dual solution associated with constraint (3b) by $\lambda_t^\star(S) \in \mathbb{R}^N$, $t \in \mathcal{T}$. Then $\lambda_{n,t}(S)$ is the LMP at bus n and time t for a given vector of storage capacity S. It is common practice in transmission markets to use these LMPs to calculate the payments from/to market participants. For instance, the payment to the generator at bus n and the (negative) payment to the load at bus n are calculated as, respectively,

$$\Pi_n^{\mathrm{G}}(S) = (\lambda_n^\star(S))^\top g_n^\star(S),\ \Pi_n^{\mathrm{L}}(S) = -(\lambda_n^\star(S))^\top d_n^\star(S).$$

Similarly, the aggregate payment to the storage at bus n is

$$\Pi_n^{\mathrm{S}}(S) = -(\lambda_n^\star(S))^\top u_n^\star(S). \tag{4}$$

B. Aggregator model

We consider a simple aggregator model: in addition to aggregating and representing the distributed storage in the transmission market, the aggregator at bus n splits the aggregate storage control signal to individual storage devices and splits the aggregate payment to individual storage owners proportionally. In particular, the payment received by individual storage owner at bus n for offering one unit of storage capacity is $\Pi_n^{\mathrm{S}}(S)/S_n$.

C. Investor model

Residing on the distribution feeder connected to each bus of the transmission grid, there are a large number of small potential investors for distributed energy storage (e.g. home owners who are interested to install residential behind-the-meter batteries). We model these small investors as a continuum, indexed by $i \in \mathcal{I}_n = [0,1]$, $n \in \mathcal{N}$. Motivated by the lumpiness of the capital and installation cost as a function of the battery capacity, we model each user's decision on storage investment as a discrete choice $s_i \in \{0,1\}$. If $s_i = 0$, the investor decides not to invest in any storage device. This requires no cost and brings no benefit so the payoff to the investor is 0. If $s_i = 1$, the investor determines to invest in one (normalized) unit of storage. The payoff to the investor $i \in \mathcal{I}_n$ in this case is $\frac{1}{S_n}\Pi_n^{\mathrm{S}}(S) + \theta_i - \kappa$, where κ is the amortized capital and installation cost of the storage device, and θ_i drawn from distribution F_n models the *outside value* of storage, i.e., the value that the storage provides to the investor in addition to the payment for participating in the transmission market. Note that θ_i can only capture values of the storage that are realized without significant interference to the storage device's ability to participate in the transmission market (e.g., the reliability value of storage that is realized in rare blackout events). In summary, the payoff to investor $i \in \mathcal{I}_n$ is

$$\pi_i(s_i, S) = \left(\frac{1}{S_n}\Pi_n^{\mathrm{S}}(S) + \theta_i - \kappa\right) s_i. \tag{5}$$

Since the only difference among investors $\mathcal{I}_n$ residing on the same bus n is that they may have different outside value θ_i, the individual investment decision is a function of the outside value θ_i for some function $\sigma_n : \mathbb{R} \mapsto \{0,1\}$:

$$s_i = \sigma_n(\theta_i), \quad i \in \mathcal{I}_n, \quad n \in \mathcal{N}.$$

Given the individual decisions on whether to invest in storage or not, the aggregate storage capacity at bus n is simply the proportion of investors $i \in \mathcal{I}_n$ who decide to invest in storage:

$$S_n = \mathbb{E}\sigma_n(\theta_i) = \int_{\mathcal{I}_n} \sigma_n(\theta_i)\,\mathrm{d}\,F_n(\theta_i). \tag{6}$$

D. Solution concept and benchmarks

To this point, we have defined the *network storage investment game*, where the set of *nonatomic* players is $\cup_{n\in\mathcal{N}}\mathcal{I}_n$, and the payoff of each player i is defined according to (5). The payoff of player $i \in \mathcal{I}_n$ depends on the *aggregate* decisions of other players who reside on both bus n and other buses, through the coupling term $\Pi_n^{\mathrm{S}}(S)/S_n$ which is determined by the solution of the multi-period economic dispatch problem (3). In this sense, the game is an aggregate game.

To understand the outcome of the game, we utilize the solution concept of Nash:

Definition 1: A vector of storage capacity $S \in \mathbb{R}^N$ constitutes a Nash equilibrium[2] if for each bus $n \in \mathcal{N}$ there exists a mapping $\sigma_n : \mathbb{R} \mapsto \{0,1\}$ such that

$$S_n = \int_{\mathcal{I}_n} \sigma_n(\theta_i)\,\mathrm{d}\,F_n(\theta_i), \tag{7a}$$

and for all $s_i \in \{0,1\}$ and $i \in \mathcal{I}_n$,

$$\lim_{\epsilon\to 0}\ \pi_i(\sigma_n(\theta_i), S + \epsilon\sigma_n(\theta_i)e_n) \geq \pi_i(s_i, S), \tag{7b}$$

where $e_n \in \mathbb{R}^N$ is the vector whose n-th entry is 1 and other entries are 0's.

In the definition above, we focus on aggregate storage capacities since the game is an aggregate game. The mapping σ_n specifies the individual investment decisions for investors on bus n. Equation (7a) ensures that the individual investment decisions are compatible with the aggregate storage capacity at each bus. Equation (7b) guarantees that there is no unilateral incentive for any investor to deviate from the decision specified by σ_n. In particular, we can interpret S as the aggregate storage capacity of all the agents except i in (7b). Then $S + \epsilon\sigma_n(\theta_i)e_n$ is the aggregate storage capacity when agent i determines to make the investment decision $\sigma_n(\theta_i)$. Since each player is infinitesimally small in our game, the impact of agent i's decision on the aggregate storage capacity is vanishingly small as $\epsilon \to 0$. We will denote storage capacity vectors satisfying conditions in Definition 1 by S^{ne}.

To better understand the characteristics of NE of the storage investment game, we will compare NE aggregate storage capacities with the following benchmarks.

1) Myopic investment: In this case, individual investors will assume themselves as price-takers. Thus they make their storage investment decisions based on the payoff function $\pi_i(s_i, 0)$. That is, they use the status-quo LMPs (i.e., LMPs

[2]Definition 1 is *not* the standard definition of Nash equilibrium for nonatomic game where the set of players consists of a continuum of small agents. Specifically, in the standard definition, (7b) is replaced by

$$\pi_i(\sigma_n(\theta_i), S) \geq \pi_i(s_i, S), \tag{7b'}$$

since each infinitesimally small agent cannot alter the aggregate storage capacity. Nevertheless, (7b') and (7b) are equivalent when the payoff function π_i is continuous with respect to the vector of storage capacities, which is the case for Section IV and VI of the paper. We have chosen (7b) instead of (7b') since when the payoff function is discontinuous (as in Section V), (7b) results in equilibrium that are consistent with that of the finite version of the game whereas (7b') does not.

for the case without any storage investment) to guide their storage investment decisions. We denote the resulting aggregate storage capacity vector by S^{myop}.

2) Monopoly investment: In this case, a monopoly represents the overall benefit of all storage investors. The monopoly solves the following optimization to determine the investment decisions to maximize the total benefit of all investors:

$$\max_{\sigma_n(\cdot),S_n,\forall n} \quad \sum_{n\in\mathcal{N}} \int_{\mathcal{I}_n} \pi_i(\sigma_n(\theta_i), S)\,\mathrm{d}\,F_n(i) \tag{8a}$$

$$\text{s.t.} \qquad S_n = \int_{\mathcal{I}_n} \sigma_n(\theta_i)\,\mathrm{d}\,F_n(\theta_i), \quad \forall n\in\mathcal{N}. \tag{8b}$$

We will denote the aggregate storage capacity vector that solves the program above by S^{mon}.

3) Social welfare: If we take the social planner's perspective, the optimal storage investment decision that maximize the total social welfare is the solution of the following optimization:

$$\max_{\sigma_n(\cdot),S_n,\forall n} \quad \sum_{n\in\mathcal{N}} \int_{\mathcal{I}_n} (\theta_i-\kappa)\sigma_n(\theta_i)\,\mathrm{d}\,F_n(\theta_i) - J(S) \tag{9a}$$

$$\text{s.t.} \qquad S_n = \int_{\mathcal{I}_n} \sigma_n(\theta_i)\,\mathrm{d}\,F_n(\theta_i), \quad \forall n\in\mathcal{N}, \tag{9b}$$

where the first term of the objective function is the total benefit received by the storage investors excluding the transfers (i.e. payments they received from the aggregators), and the second term is the (optimal) operation cost the grid given the storage capacity vector. Denote a vector of storage capacities that solves (9) as S^{sw}. We will say a NE supports social welfare if it achieves the same objective value as S^{sw}.

Albeit our solution concept and benchmarks are conceptually simple, it is challenging to characterize them explicitly in general settings due to the coupling of storage investment decisions induced by the power network and the payment function for each investor i defined implicitly through the solution of the multi-period economic dispatch problem (3). To develop qualitative insights, we first develop structural properties of the coupling term in Section III and then analyze and explicitly characterize the network storage investment game in two specific settings in Section IV and Section V.

III. Structural Results

As a first step to explicitly characterize the NE and the benchmark solutions, we provide the following characterization to the coupling term in the payoff function of the individual investors. Due to the space limit, all proofs are omitted except that we sketch the proof of Theorem 3 (the most technical result in the paper) in the appendix.

Proposition 1: For each bus $n\in\mathcal{N}$,

$$\frac{1}{S_n}\Pi^{\text{S}}_n(S) = \sum_{t=1}^{T}\nu^\star_{n,t}(S) = \sum_{t=1}^{T}\left(\lambda^\star_{n,t+1}(S)-\lambda^\star_{n,t}(S)\right)_+,$$

where $\nu^\star_n(S)$ is the optimal dual solution associated with constraint $Lu_t \le S_n\mathbf{1}$ in (3), $\lambda^\star_{n,T+1}(S) := 0$ and $(x)_+ := \max(x,0)$.

The expression above provides a simple and intuitive way to calculate the payment to investors for each unit of storage based on LMP differences across consecutive time periods.

One challenge in obtaining the benchmark solutions (8) and (9) is the fact that these are infinite dimensional optimizations (i.e., optimal control problems). The following structural result converts the infinite dimensional problems into finite dimensional optimizations.

Proposition 2: An optimal individual storage investment policy $\sigma_n(\theta_i)$ for problem (8) (or (9)) takes the form of

$$\sigma_n(\theta_i) = \begin{cases} 1 & \text{if } \theta_i \ge \bar{\theta}_n, \\ 0 & \text{otherwise,} \end{cases}$$

for some $\bar{\theta}_n\in\mathbb{R}$, $n\in\mathcal{N}$.

In other words, optimal individual investment policies for the monopoly investment benchmark and the social welfare maximization benchmark are threshold policies based on the outside values of the users, which are the only differentiating factor among different users residing on the same bus. This dramatically simplifies problem (8) and (9) since we can now optimize over a vector of thresholds instead of all possible individual investment policies.

IV. Uncongested Network

We start by considering a single bus network ($N=1$) with two time periods ($T=2$)[3]. The single bus model has been used in storage investment studies [4], [5] where power network constraints are not considered or the network is unlikely to be congested. In those studies, it is often the case that NE is unique and supports social welfare. Our goal of this section is to examine whether such properties also hold in our model with a continuum of investors and without the price-taker condition often assumed in prior studies [4], [5].

In particular, we consider the case depicted by the generalized network flow diagram [13], [14] in Fig. 2. In this case, there is a supply at time 1 and a demand at time 2. We assume the generation cost[4] is

$$c(g) = \frac{1}{2}\alpha g^2 + \beta g, \tag{10}$$

and the benefit function of the demand is

$$b(d) = \gamma d, \tag{11}$$

where $\alpha>0$ and $b'(0)-c'(0)>\kappa$. We also assume that the storage edge in the generalized flow network is congested (i.e., $u^\star_t = S$ for $t=1$ and $u^\star_t=-S$ for $t=2$) so that there will be a price differential across the two periods[5] and $\Pi_i(S)>0$.

Equipped with the structural results in Section III, we are ready to provide explicit solutions and benchmarks for the storage investment game in the uncongested network case.

[3] We will omit bus index when there is no source of confusion.

[4] It suffices to have one of functions $c(g)$ and $-b(d)$ be strictly convex so that the storage capacity impacts the LMP differential across the two time period. Replacing the quadratic functions to be more general differentiable convex functions or having both functions to be strictly convex does not change our qualitative conclusions in this and the next sections.

[5] See e.g. [8] on how LMP differentials relate to the storage congestion pattern.

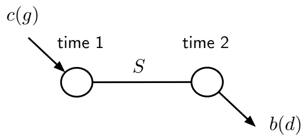


Fig. 2: Generalized flow diagram for the uncongested network case ($N = 1$ and $T = 2$). Each node represents a bus for a time period. At each node, an arrow pointing towards the node represents a supply and an arrow pointing from the node represents a demand. An edge connecting two nodes within in same time period is a transmission line and an edge connecting two nodes in different time periods is a storage.

Lemma 1 (Benchmarks, uncongested network): For the single bus and two period setup with the cost of generation specified by (10) and the benefit of consumption specified by (11), the following statements hold.

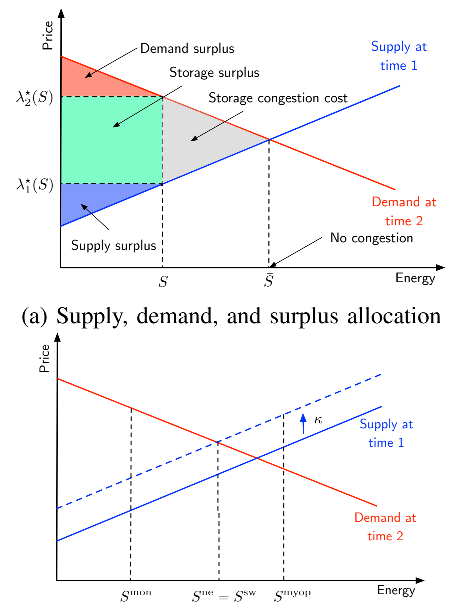
1) The myopic storage investment capacity is
$$S^{\mathrm{myop}} = 1 - F(\kappa + \beta - \gamma),$$
where F is the cumulative distribution function of outside value θ_i at the bus.
2) The monopoly storage investment capacity is the solution of the fixed point equation
$$S^{\mathrm{mon}} = 1 - F(2\alpha S^{\mathrm{mon}} + \kappa + \beta - \gamma).$$
3) The social welfare maximizing storage investment capacity is the solution of the fixed point equation
$$S^{\mathrm{sw}} = 1 - F(\alpha S^{\mathrm{sw}} + \kappa + \beta - \gamma).$$

Theorem 1 (NE, uncongested network): Under the same conditions of Lemma 1, the following statements hold.

1) NE exists and is unique.
2) $S^{\mathrm{mon}} \le S^{\mathrm{ne}} = S^{\mathrm{sw}} \le S^{\mathrm{myop}}$.

Lemma 1 and Theorem 1 can be easily digested through Fig. 3 when $\theta_i \equiv 0$. In Fig. 3a, the upward-sloping supply curve is derived from the cost function of generation $c(\cdot)$ in time period 1; the downward-sloping demand curve is derived from the benefit function $b(\cdot)$ of demand[6] in time period 2. For a fixed storage capacity S, we can read the LMPs in time periods 1 and 2 from the supply curve and demand curve, respectively. Then the surplus received by the supply, demand, and storage can be read from the correspondingly labeled areas. The grey triangle is then the surplus not achieved due to the fact that the storage capacity is not large enough and so the storage is congested at the optimal solution of the multi-period economic dispatch problem. Because of the per-unit capital and installation cost of storage κ, the actual cost for supplying one unit of energy in time period 1 to serve the demand in time period 2 is larger than just the generation cost represented by the original supply curve. In Fig. 3b, this is captured by lifting the supply curve by κ. As a result, the intersection of the demand curve and the effective supply curve (i.e. the dashed blue line) is both the social welfare optimal storage investment and the NE. The fact that NE supports social welfare in this case can be understood as a consequence of *competition* among an infinite number of small players – more people investing in storage increases the aggregate storage capacity which in turn reduces the price differential across two time periods and therefore reduces the revenue of each investor. The monopoly decision under-invests because a smaller storage keeps a price differential larger than κ, maximizing the profit all the storage investors can make. This however is done at the cost that the supply surplus and demand surplus are at lower levels than that corresponding to the social welfare optimal capacity. The myopic decision over-invests because it does not account in the effect that the price differential diminishes when more people invest in storage.

[6]The figure depicts a more general case; for our specific linear benefit function (11), the demand curve is a constant function.



(a) Supply, demand, and surplus allocation

(b) NE and benchmarks for uncongested network

Fig. 3: Demonstration of NE and benchmarks for the case without outside value ($\theta_i \equiv 0$).

In the case with nonzero and stochastic outside value θ_i, the situation is not as simple as demonstrated in Fig 3 and so the solutions are often defined via fixed point equations involving the distribution of the outside value F. However, we have similar qualitative conclusions as discussed for Fig. 3.

V. Complementary Storage Investment

Although our results in the previous section on uncongested network deliver only positive messages (NE exists, is unique, and supports social welfare), they may not remain valid for more general settings. In this section, by extending Joskow and Tirole's example for transmission capacity investment [10] to incorporate multiple periods and storage, we examine a setting where storage capacity at one bus perfectly complements the storage capacity at another bus, so only if both buses invest in storage no benefit for the grid will be derived from the storage investment. The same example has been shown to be a hard instance for the centralized storage placement problem in prior work [8].

Consider a two-bus three-period setting as demonstrated by the generalized network flow diagram in Fig. 4. In this case, the transmission line connecting the two buses is not available in time period 1 and 3 (e.g., because the line is on a scheduled maintenance in these periods). There is a supply at bus 1 available in time period 1 with cost function $c(g)$ as defined in (10) and a demand at bus 2 in time period 3 with benefit function $b(d)$ as defined in (11), where $\alpha > 0$ and $b'(0) - c'(0) > 2\kappa$. Given the topology of the generalized flow network, the only flow path to send energy from the supply to demand is to first use the storage at bus 1, then use the transmission line in time period 2, and finally use the storage at bus 2.

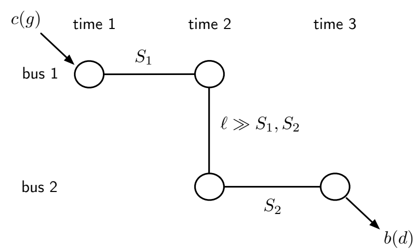


Fig. 4: Generalized flow diagram for the complementary storage investment case ($N = 2$ and $T = 3$). See the caption of Fig. 2 for explanations of the figure.

To simplify, we assume the transmission capacity is significantly larger than the storage capacities $\ell \gg S_1, S_2$ and the storage capacities at the solutions are such that at least the storage at one bus is congested (otherwise the storage capacities can be reduced without reducing the benefit of storage for the grid). Since the transmission line is not congested in time 2 and so the LMPs at both buses in time 2 are identical, we will denote the LMPs in three time periods by $\lambda_t^\star(S)$, $t = 1, 2, 3$. Furthermore, we assume symmetry between two buses: the distributions of outside values on the two buses are the same $F_1 = F_2 = F$.

Under these assumptions, we can focus on symmetric solutions ($S_1 = S_2$) for both NE and benchmarks. When $S_1 = S_2$, the LMP at time period 2 is not uniquely determined as the multi-period economic dispatch problem is degenerate. We resolve this issue by defining

$$\lambda_2^\star(S) = \lambda_1^\star(S) + \eta(\lambda_3^\star(S) - \lambda_1^\star(S)),$$

and pick[7] $\eta = 1/2$. As a consequence, the aggregate payment for each unit of storage capacity on both buses is

$$\left[\frac{\Pi_1^{\mathrm{S}}(S)}{S_1}, \frac{\Pi_2^{\mathrm{S}}(S)}{S_2}\right] = \begin{cases} (\lambda_3^\star(S) - \lambda_1^\star(S)) \cdot [1, 0], & S_1 < S_2, \\ (\lambda_3^\star(S) - \lambda_1^\star(S)) \cdot \left[\frac{1}{2}, \frac{1}{2}\right], & S_1 = S_2, \\ (\lambda_3^\star(S) - \lambda_1^\star(S)) \cdot [0, 1], & S_1 > S_2. \end{cases}$$

That is, the bus with strictly smaller storage capacity takes all the surplus; when two buses have the same identical storage capacity the surplus is shared equally between them.

We state the benchmark solutions and characterize the set of NE in the following results.

[7]Changing η to other values in $(0, 1)$ does not change our qualitative messages.

Lemma 2 (Benchmarks, complementary storage): For the two bus and three period setup with the cost of generation specified by (10) and the benefit of consumption specified by (11), the following statements hold.

1) The myopic storage investment capacity vector is $(S^{\mathrm{myop}}, S^{\mathrm{myop}})$, where
$$S^{\mathrm{myop}} = 1 - F(\kappa).$$
2) The monopoly storage investment capacity vector is $(S^{\mathrm{mon}}, S^{\mathrm{mon}})$, where S^{mon} is the solution of the fixed point equation
$$S^{\mathrm{mon}} = 1 - F\left(\alpha S^{\mathrm{mon}} + \kappa + \frac{1}{2}(\beta - \gamma)\right).$$
3) The social welfare maximizing storage investment capacity vector is $(S^{\mathrm{sw}}, S^{\mathrm{sw}})$, where S^{sw} is the solution of the fixed point equation
$$S^{\mathrm{sw}} = 1 - F\left(\frac{1}{2}\alpha S^{\mathrm{sw}} + \kappa + \frac{1}{2}(\beta - \gamma)\right).$$

Theorem 2 (NE, complementary storage): Under the same conditions of Lemma 2, the following statements hold.

1) NE exist but are not unique.
2) Any storage capacity vector $(S^{\mathrm{ne}}, S^{\mathrm{ne}})$ constitutes an NE if $S^{\mathrm{ne}} \in [S^{\mathrm{myop}}, S^{\mathrm{sw}}]$.

When $\theta_i \equiv 0$, these solutions are depicted in Fig. 5. Since serving the demand in time 3 with the supply in time 1 requires building storage on both buses, the supply curve is lifted by 2κ in this case. The myopic solution under-invests (in fact $S^{\mathrm{myopc}} = 0$ if $\theta_i \equiv 0$) because without anticipating storage investment on the other bus, myopic investors will expect no payment from the grid. Similar to the discussion in the previous section, the monopoly decision also under-invests to raise the profit for storage investors at the cost of other grid market participants.

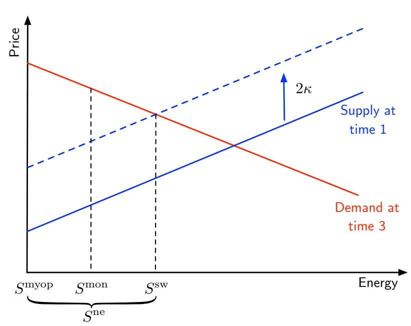


Fig. 5: Demonstration of NE and benchmarks for the case without outside value ($\theta_i \equiv 0$).

The major qualitative difference between the results in this section and that in the previous section is that in the complementary storage case NE are no long unique and may not support social welfare. This may be understand analytically by noticing: (i) each investor takes a 0-or-1 decision, (ii) the payoff function of each investor is discontinuous in the storage capacity at the NE so even an infinitesimally small investor switch his decision can dramatically change the allocation of surplus between the two buses, and therefore

(iii) there will be a non-trivial range of aggregate storage capacities (which correspond to different portions of the population) that can be NE since investing players do not have incentive to switch to not investing (which leads to zero payoff), and non-investing players do not have incentive to switch to investing (which leads to a higher storage capacity for his bus and zero payment from the grid). Intuitively, due to the structure of the payment function $\Pi_n^{\mathrm{S}}(S)$, the investors on each bus are incentivized to have a smaller aggregate storage capacity than the other bus. This effect is known to lead to inefficient NE in the transmission investment problem [10].

VI. General Network

Analysis in the previous section suggests that the positive results in the uncongested network case may not hold in general and there may be NE that do not support social welfare. This indicates that the incentives as outlined in Section II may need to be adjusted e.g. by regulators if a social welfare optimal distributed storage investment level is desired.

In the remainder of this section, we provide analytical results that may suggest directions for such regulatory interventions. This is done by characterizing the set of NE and establishing that each NE supports social welfare for *general networks*, provided that we include the following two modifications/assumptions in our model:

A1 *Continuous investment:* Each investors may make a continuous decision on how much storage to invest, i.e., $s_i \in [0, 1]$ for all $i \in \mathcal{I}$.

A2 *Strict convexity:* For each bus $n \in \mathcal{N}$ and time $t \in \mathcal{T}$, the cost of generation $c_{n,t}(\cdot)$ and the negative benefit of demand $-b_{n,t}(\cdot)$ are strictly convex.

A1 shifts the focus of the model on the lumpiness of individual storage investment to the fact that investors usually do have a range of choices in terms of the storage capacity to be installed (e.g. the capacity of Tesla Powerwall can be customized). Modeling the capital and installation cost by κs_i as done in (5) now becomes less accurate as it does not capture the fixed cost of storage systems (e.g., cost of inverter and installation). Another possible interpretation of **A**1 is that in the aggregator business model, we allow each user to only offer a part of the storage capacity to the aggregator if the user has decided to invest and install a behind-the-meter storage. In this setting, if $s_i > 0$ (i.e., the user invested in storage), then real value s_i denotes how much storage capacity the user is willing to release to the aggregator, with the remaining capacity reserved for the user's own use. In view of discussions in the complementary storage setting, we observe that users are indeed incentivized to offer a smaller capacity to the aggregator in order to result in a smaller aggregate storage capacity in his own bus which in turn leads to a higher payment from the transmission market.

For power generation, **A**2 is often valid as strictly convex costs model the increasing marginal cost of power generation [15]. The justification of **A**2 for the demand side is less clear. In particular, if the demand at some bus is inelastic, then such an assumption does not hold. When we assume the demand is always elastic, **A**2 amounts to diminishing marginal return for consuming more electricity, which is economically sensible. Alternatively, we can consider **A**2 as a consequence of certain supply/demand function bidding rule implemented by the transmission market operator. In particular, if all the participants of the transmission market is required to bid a linear supply (or demand) function with a finite non-zero slope, then the corresponding cost (or negative benefit) function in the multi-period economic dispatch problem (3) naturally is strictly convex and quadratic. However, an issue with this interpretation is that a larger discrepancy between the cost/benefit functions in the multi-period economic dispatch problem and the actual cost/benefit functions is expected when the supply/demand function is restricted to a smaller function class [16], [17].

With these two modifications in place, we have the following main result:

Theorem 3: Under **A**1 and **A**2 and for general networks, all NE support social welfare.

Theorem 3 does not guarantee the uniqueness of NE. Indeed, consider a network with two buses connected by a transmission line that is essentially uncapacitated and assume there is no outside value so $\theta_i \equiv 0$. Clearly, if $(S_1^{\mathrm{ne}}, S_2^{\mathrm{ne}})$ constitutes an NE, then any vector in the set $\{(S_1, S_2) \in \mathbb{R}_+^2 : S_1 + S_2 = S_1^{\mathrm{ne}} + S_2^{\mathrm{ne}}\}$ is also an NE. Theorem 3, nevertheless, asserts that all NE will lead to the same level of social welfare as measured by the objective function of (9). Furthermore, any NE storage capacity vector S^{NE} also coincides with a solution of the social welfare maximization program (9) and therefore is social welfare optimal.

VII. Conclusions

We formulate and analyze a network storage investment game where the distributed storage capacities at each distribution feeder are aggregated to participate in the wholesale electricity market. By explicitly characterizing the set of NE and comparing with several benchmark solutions, we show that (i) for an uncongested network, NE is unique and supports social welfare, and (ii) in a complementary storage investment setting, NE are not unique and may not support social welfare. We then show that if we include two modifications for the model, it is guaranteed that NE support social welfare even for general power networks. With caveats discussed in Section VI, these modifications may be realized by changing the aggregator business model – allowing each storage owner to offer only a portion of his storage for grid use, and by restricting the bidding format in the wholesale electricity market – requiring each wholesale market participants (i.e., supplier or consumer) to bid a linear supply or demand function with a finite non-zero slope.

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APPENDIX I
PROOF SKETCH OF THEOREM 3

Our proof strategy is outlined in Fig. 6. In words, for each S^{NE}, we consider a finite-player version of the game, where there are m investors on each bus. We generate their outside value θ_i, $i \in \mathcal{I}_n$, by creating m independent samples from the distribution F_n. In the finite-player game, we consider two solution concepts: a storage capacity vector that solves a set of *mean-field equations*, denoted by $S^{\mathrm{MFE}(m)}$, and a storage capacity vector that solves an *empirical social welfare optimization* problem, denoted by $S^{\mathrm{sw}(m)}$. We show in the following that *1)* $S^{\mathrm{MFE}(m)}$ approximates S^{NE} when $m \to \infty$, *2)* $S^{\mathrm{sw}(m)}$ approximates S^{sw} when $m \to \infty$, and *3)* for each m, $S^{\mathrm{MFE}(m)}$ and $S^{\mathrm{sw}(m)}$ coincide.

$$\begin{array}{ccc} S^{\mathrm{MFE}(m)} & \xrightarrow[m\to\infty]{\text{a.s.}} & S^{\mathrm{NE}} \\ \| & & \\ S^{\mathrm{sw}(m)} & \xrightarrow[m\to\infty]{\text{a.s.}} & S^{\mathrm{sw}} \end{array}$$

Fig. 6: Strategy for proving Theorem 3. The convergences should be understood in a loose term – the precise meanings can be found in the discussions below.

1) “$S^{\mathrm{MFE}(m)} \xrightarrow[m\to\infty]{\text{a.s.}} S^{\mathrm{NE}}$”: The first step is to establish the following technical lemma.

Lemma 3: Under **A**1 and **A**2, for each $n \in \mathcal{N}$, $\Pi_n^S(S)/S_n$ is locally Lipschitz continuous with respect to S.

Lemma 3 follows from Theorem 2 of [18] under Linear Independent Constraint Qualification (LICQ), i.e., constraints of (3) that are binding at the solution are linearly independent. Due to the special structure of (3b)-(3d), we can actually show that Lemma 3 holds even without LICQ conditions. This is done by properly removing some binding *inequality* constraints from (3c) until LICQ holds in a way that does not change $\lambda_n^\star(S)$.

We then consider the following *mean-field equations*:

$$\begin{aligned} &\pi_i(\sigma_n(\theta_i), S) \geq \pi_i(s_i, S),\ \forall s_i \in [0,1], i \in \mathcal{I}^m, \\ &S_n = \frac{1}{m}\sum_{i\in\mathcal{I}_n^m} \sigma_n(\theta_i), \qquad \forall n \in \mathcal{N}, \end{aligned} \tag{MFE(m)}$$

where $\mathcal{I}_n^m$ is the set of m players residing on bus $n \in \mathcal{N}$, and $\mathcal{I}^m = \cup_{n\in\mathcal{N}}\mathcal{I}_n^m$. We denote a solution to MFE(m) by $S^{\mathrm{MFE}(m)}$.

Utilizing the Lipschitz continuity of the payment function (Lemma 3) and the law of large numbers, we have the following convergence result.

Lemma 4: Under the assumption of Lemma 3, for any S^{NE}, there exists a sequence $\{S^{\mathrm{MFE}(m)} : m \in \mathbb{Z}_+\}$ such that $S^{\mathrm{MFE}(m)}$ is a solution to MFE(m) for $m \in \mathbb{Z}_+$, and $S^{\mathrm{MFE}(m)}$ converges to S^{NE} almost surely as $m \to \infty$.

2) “$S^{\mathrm{sw}(m)} \xrightarrow[m\to\infty]{a.s.} S^{\mathrm{sw}}$”: Consider the following empirical social welfare optimization problem:

$$\max \quad \sum_{n\in\mathcal{N}}\sum_{i\in\mathcal{I}_n^m} \frac{1}{m}(\theta_i - \kappa)\sigma_{n,i} - J(S) \tag{12a}$$

$$\text{s.t.} \quad S_n = \frac{1}{m}\sum_{i\in\mathcal{I}_n^m}\sigma_{n,i}, \quad \forall n \in \mathcal{N}, \tag{12b}$$

where the decision variables are $\sigma_{n,i} \in [0,1]$, $n \in \mathcal{N}$, $i \in \mathcal{I}_n^m$, and $S_n \in \mathbb{R}_+$, $n \in \mathcal{N}$. To simplify, we can eliminate the variable S_n, $n \in \mathcal{N}$ and constraint (12b) by substituting S_n in the objective function with $\frac{1}{m}\sum_{i\in\mathcal{I}_n^m}\sigma_{n,i}$. We then observe that (12) is a sample average approximation [19] to (9). Denote the optimal value of (12) by V^m and the optimal value of (9) by $V^\star$. We then have the following result:

Lemma 5: Under the assumption of Lemma 3, $V^m \to V^\star$ almost surely as $m \to \infty$.

3) “$S^{\mathrm{MFE}(m)} = S^{\mathrm{sw}(m)}$”: By constructing an optimization problem based on MFE(m) and connecting it to (12) via duality theory [20], we have the following theorem:

Theorem 4: Under the assumption of Lemma 3, any S^m that satisfies MFE(m) is an optimal solution to (12).