Estimation with Generalized-T Distribution noise model

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Supported by:
Funded by: NATIONAL RESEARCH FOUNDATION

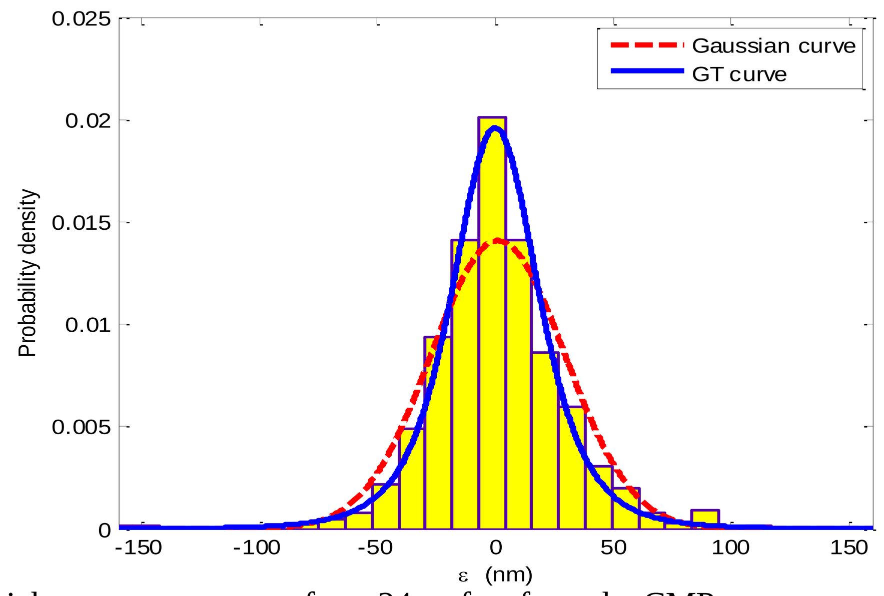
Motivation

- Noise can be non-Gaussian.
- Least-squares estimation assumes Gaussian noise.
- Least-squares estimation is sensitive to outliers.
- Use robust statistics to handle non-Gaussian noise and outliers.

Main Objectives

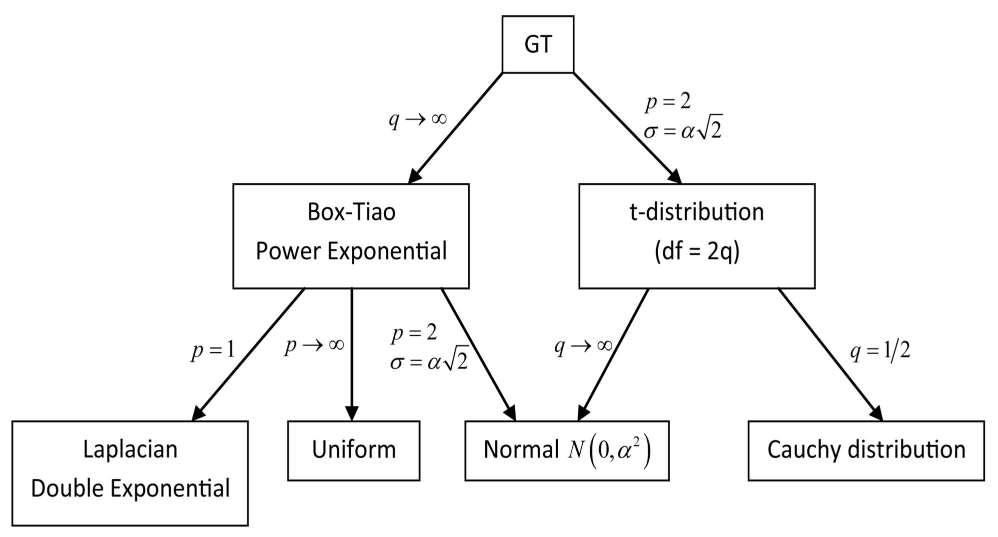
- Develop a parameter and state estimator system that can deal with non-Gaussian noise.
- Develop a framework using the Influence Function to analyze the properties of the estimator.

Non-Gaussian measurement data



Thickness measurement from 24 wafers from the CMP process.
The maximum likelihood criterion was used to fit the Gaussian distribution (dotted) and GT distribution (solid) to the thickness measurements.

Generalized-T Distribution



Different choices of the GT distribution $f(\varepsilon) = \dfrac{p}{2\sigma q^{1/p}\beta(1/p,q)\left(1+\dfrac{|\varepsilon|^p}{q\sigma^p}\right)^{q+1/p}}$ shape parameters p and q can give different well-known distributions.

Methodology 1 - Parameter Estimation

Consider the linear in parameter model:

$$y(k) = \phi(k)^T\theta + \varepsilon(k)$$

where $\phi(k) = [\phi_1(k),\ldots,\phi_n(k)]^T$ are known.

$\theta = [\theta_1,\ldots,\theta_n]^T$ are to be estimated.

$k = 1\ldots N$ is the sampling instance.

To obtain the maximum likelihood estimate of the initial condition θ, we minimize the cost function

$$J = -\sum_{k=1}^{N}\ln\left(f(y(k)-\theta(k)^T\theta)\right)$$

where $\varepsilon(k)$ are modeled as GT.

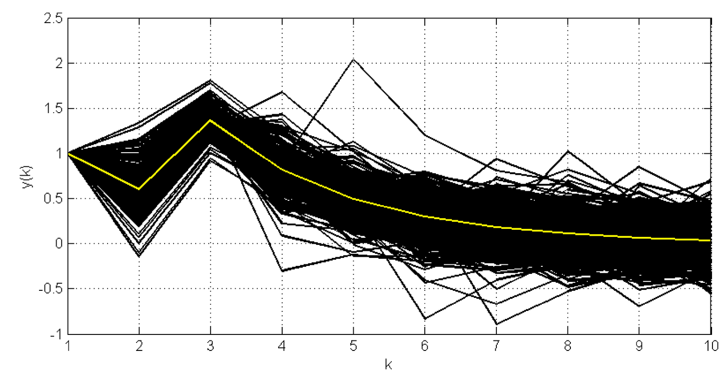
The AR model with outlier

Consider the autoregressive (AR) model:

$$y(k) = ay(k-1) + \varepsilon(k)$$

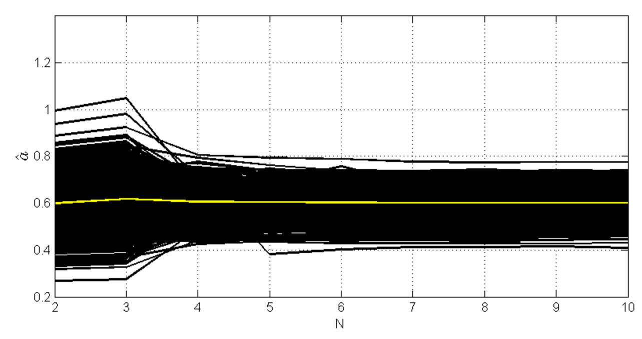
with $\varepsilon(k)$ belongs to the following distribution

$$\varepsilon(k) = \begin{cases} \delta(\varepsilon_1) & k = k_1 \\ f(\varepsilon) & k \neq k_1 \end{cases}$$

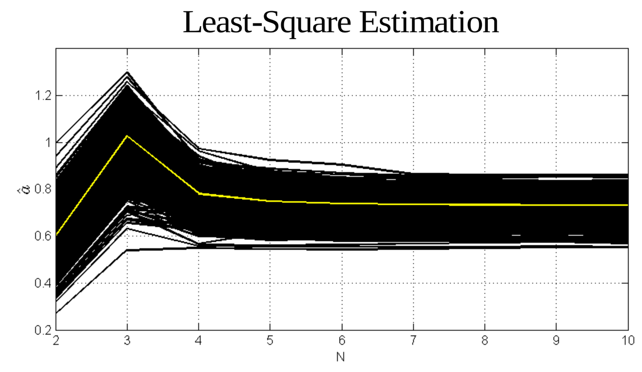


Estimation Results

Estimator with GT noise model



Least-Square Estimation



Methodology 2 - State Estimation

Consider the discrete model:

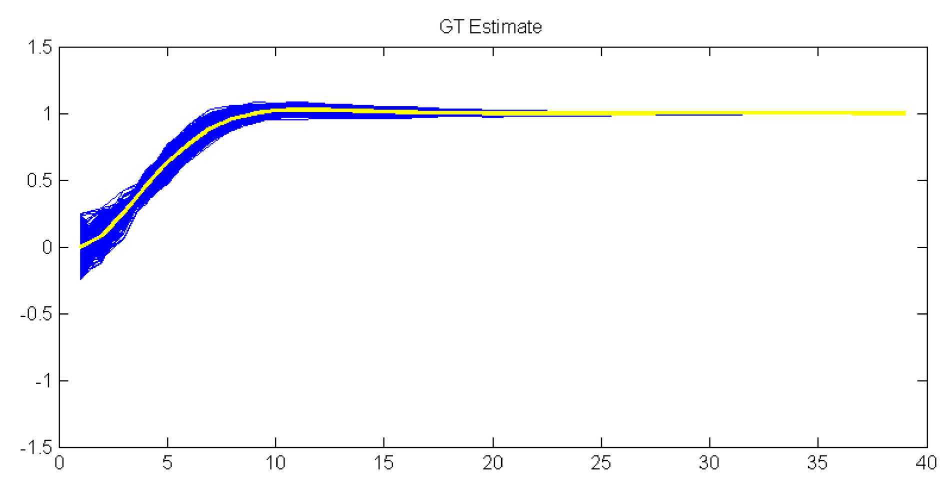
$$Y(z) = \frac{B(z)}{A(z)}U(z) + E(z)$$

The recursive algorithm:

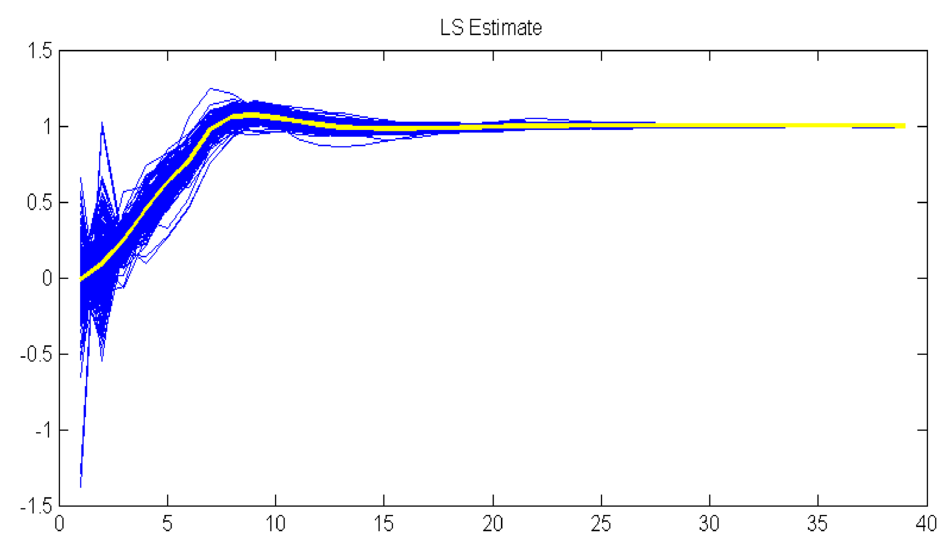
$$\hat{x}(N\,|\,N-1) = A\hat{x}(N-1\,|\,N-1) + bu(N-1)$$
$$\varepsilon(N) = y(N) - c\hat{x}(N\,|\,N-1)$$
$$z(N) = \rho(\varepsilon(N))$$
$$P(N) = (P(N-1)^{-1} + \phi(N)^T\phi(N))^{-1}$$
$$\Delta\hat{x}(N\,|\,N) = A^{N-1}P(N)\phi(N)^T z(N)$$
$$\hat{x}(N\,|\,N) = \hat{x}(N\,|\,N-1) + \Delta\hat{x}(N\,|\,N)$$

Closed-loop Control

Estimator with GT noise model



Least-Square Estimation



Impact / expected result

- Robust estimation of parameters and states of a building.
- Demonstrate result on building data.

Future Goals

- Extend the methodology to handle a wider range of process model.
- Reduce computation.